Heterogeneity, Decentralized Trade, and the Long-run Real Effects of Inflation

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Abstract

Real effects of long-run inflation are studied in a standard matching model of money. Depending on the underlying policy, inflation can increase output but decrease *ex ante* social welfare or can do the opposite. Inflation makes the distribution of wealth more concentrated at the very top. When facing a potential policy change, the poor are much more sensitive to which policy may be adopted than the rich. Decentralized trade plays a critical role in these findings by amplifying the individual risk on labor income earning.

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Key Words: Heterogeneity, Bilateral Bargaining, Inflation, Inequality, Welfare

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1 Introduction

This paper examines the long-run real effects of inflation with heterogeneous agents. It is not new that inflation would have redistribution effects by different channels. Our paper concerns a physical environment in which an individual earns and spends his labor income through a decentralized-trade process; such an environment is realistic and, it is of interest because how much the individual can adjust his wealth status depends on how much his trading partner is willing to adjust. Also, given wealth heterogeneity, it is important to distinguish different sorts of inflation that redistribute wealth by different manners; this is an essential point of Wallace (2014). Therefore, our research question is how quantitatively a decentralized-trade process may affect the influences of different inflationary policies on output, the wealth distribution, and welfare. We consider output because it is arguably the most attention-drawing macro aggregate, the wealth distribution because monetary policy is related to the growing inequality by some public opinion,[1] and welfare because a widespread narrative says that inflation hurts poor people more than rich.[2]

Our paper is based on an off-the-shelf model, the familiar model of Trejos and Wright (1995) and Shi (1995) with general individual money holdings. Having anonymous agents trade in pairwise meetings, this basic model of the New Monetarist economics provides a solid microfoundation for money as a medium of exchange, in which who trades with whom and how the trade is conducted are explicitly described. Populated with heterogeneous agents, the model resembles much of the Bewley model, the workhorse model for studying inequality. In a canonical Bewley model (see, e.g., İmrohoroğlu 1992), each agent adjusts his wealth status on a centralized spot market; in the Trejos-Wright-Shi model, each agent does so by trading with his partner in a pairwise meeting.

In our basic model, we borrow from Wallace (2014) an abstract program which

[1] The opinion seems to reach central bankers; e.g., see Bernanke (2015), Bullard (2014), and Constâncio (2017).

[2] As documented by Easterly and Fischer (2001), poor people are more concerned about inflation.

repeatedly makes a regressive or progressive transfer of money to agents. We select a value of the buyer's weight in surplus-sharing for the parameterized model that generates the markup value commonly used in related studies. We find that (a) both regressive and progressive transfers stretch the wealth distribution (i.e., increase inequality) with respect to the zero-transfer benchmark; (b) stretching the distribution has a significant and positive effect on output, as long as overall incentives to produce are maintained; (c) only regressive transfers can maintain overall incentives to produce, thus increasing output significantly; and (d) regressive transfers decrease *ex ante* social welfare, while progress transfers may increase it. Finding (b) is in line with a key finding by Jin and Zhu (2019) for one-shot transfers in the same model. One-shot transfers alter the wealth distribution but barely alter incentives to produce. Repeated transfers alter both, and findings (a), (c) and (d) are all related to incentives.

The key to understanding incentives is the endogenized aversion to risk on wealth embodied in the indirect utility function, i.e., the value function, reflecting the individual risk on consumption and production induced by risk on wealth. Taking away an agent's wealth more when he is poorer, a regressive transfer increases the individual risk as if applying a concave transformation to the value function, a transformation that maintains overall incentives to produce; a progressive transfer does the opposite and dilutes overall incentives. The progressive transfer paradoxically stretches the distribution because of a general-equilibrium effect due to the reduction in the individual risk—agents dramatically increase their expenditures. The regressive transfer actually discourages agents to spend but the magnitude is much less dramatic, which may be understood on the basis that absent any transfer, the individual risk already sufficiently restrains spending.

Replacing decentralized trade with centralized trade on spot markets (as in the Bewley model) greatly reduces the individual risk. As such, it much reduces the degree by which a transfer alters the distribution, thus reducing the output-increase potential (if the output does increase); it also allows a regressive transfer to improve *ex ante* welfare and output at the same time. Compared with centralized trade, decentralized trade amplifies the individual risk at the benchmark mainly through the induced risk on production. While the risk-amplifying degree generally decreases as the buyer's surplus-sharing weight goes down, findings (a)-(d) above are valid for a range of the buyer's weights consistent with a wide range of the markup values. In summary, the real effects of inflation depend on the underlying policy; for each sort of policy, its real effects may differ under decentralized and centralized trade because earning labor income through bilateral bargaining can make an individual much more averse to risk on wealth than earning labor income from a competitive market. These are the very key lessons from our study.

Plausibly, an inflation policy in reality is hybrid in that it is neither (purely) regressive nor progressive. To extend our study to hybrid policies, we add government bonds to the basic model. When all injected money is used to finance interests on bonds, the inflation policy is regressive. Therefore, the government can run a class of hybrid policies according to the individual purchasing of bonds. To make a focus, we concentrate on a class of hybrid policies that deliver the same *ex ante* welfare as the zero-inflation policy. There are two notable consequences of the interaction between the progressive and regressive characteristics of these policies. One is that the progressiveness of a policy increases with inflation, limiting the room for inflation to increase output. Another pertains to the scenario that people in a steady state face a potential rise in inflation. We find that the poor favor a progressive policy, the rich favor a regressive policy, the poor are much more sensitive to which policy is adopted, and a hybrid policy can be attractive to society because it better balances the demands from the two sides.

The rest of the paper is organized as follows. We describe the basic model in section 2 and report the findings of quantitative analysis in sections 3 and 4. The model with nominal bonds is studied in section 5. Section 6 discusses the related literature. Section 7 concludes.

2 The basic model

Time is discrete, dated as $t \geq 0$. There is a unit mass of infinitely lived agents and a durable and intrinsically useless object, called money. Money is indivisible and, without loss of generality, let its smallest unit be 1; the initial money stock is M; there is a finite but arbitrarily large upper bound B on the individual money holdings; and the initial distribution of money π_0 is public information.

Each period t comprises two stages, 1 and 2. At stage 1, the government transfers money to agents in the form of lotteries; for an agent holding m units of money at the start of the period, a lottery is a probability measure on the set $\{0, ..., B-m\}$ such that the measure of x is the probability that the agent receives x units of money from the government. Following Wallace (2014), we characterize a transfer policy or, simply, a transfer, by a pair of parameters $(C_0, C) \in \mathbb{R} \times \mathbb{R}_+$: the lottery specified by the transfer for the agent holding m has a mean $z(m)$ equal to $\min\{\max\{0, C_0 + C \cdot m\}, B - m\}$ and the minimal variance (which is obtained when the support of the lottery is the two integers neighboring $z(m)$ if $z(m) \notin \mathbb{Z}$ and is $z(m)$ otherwise). The transfer is *regressive* if $C_0 < 0$ and *progressive* if $C_0 > 0$; it is helpful to note that the potential real effects of the transfer come from the component C_0.[3]

At stage 2, each agent has an equal chance of being a buyer or a seller. Following the type realization, each seller is randomly matched with a buyer. In each pairwise meeting, the seller can produce a good only consumed by the buyer. The good is divisible and perishes at the end of the period. By exerting l units of the labor input, each seller can produce l units of goods. A trading outcome in the meeting is a lottery on the feasible transfers of goods and money. If the seller exerts l units of the labor input, his disutility is

$$c(l) = l^{1+1/\eta} / (1 + 1/\eta), \ \eta > 0. \tag{1}$$

[3]When money is divisible and $B = \infty$, the transfer is purely proportional and has no real effect with $C_0 = C$.

If the buyer consumes y units of goods, his period utility is

$$u\left(y\right)=\left[\left(y+\omega\right)^{1-\sigma}-\omega^{1-\sigma}\right]/\left(1-\ \sigma\right),\ \sigma>0, \tag{2}$$

where ω is a small positive number (which keeps the buyer's reservation value well defined). Each agent can observe his meeting partner's money holdings, and the trading outcome in the meeting is determined by the weighted egalitarian solution of Kalai (1977),[4] in which the buyer's share of surplus is θ. Without loss of generality, we represent a generic trading outcome by the pair (y,μ), meaning that the seller transfers $y\geq 0$ units of goods and the buyer pays $d\in\{0,\ldots,\min(m^b,B-m^s)\}$ units of money with probability $\mu(d)$.[5]

At the end of date t, each unit of money independently disintegrates with the probability $\delta_t=1-M_t/M_t^+$, where M_t and M_t^+ are the stocks of money before and after the stage-1 transfer at period t, respectively; this disintegration turns the money stock back to M_t and implies $M_t=M$, all t.[6] Each agent maximizes his expected utility with a discount factor $\beta\in(0,1)$.

To describe equilibrium conditions at period t, let $v_{t+1}(m)$ be the value for an agent holding m units of money at the start of $t+1$ and $\pi_t(m)$ be the proportion of agents holding m units of money at the start of t. Given the distribution π_t, the proportion of agents holding m units of money immediately following the stage-1 money transfer is

$$\hat{\pi}_t\left(m\right)=\sum_{m'}\lambda_t\left(m,m'\right)\pi_t\left(m'\right), \tag{3}$$

where $\lambda_t\left(m,m'\right)$ is the proportion of agents with m' units of money receiving $m-m'$

[4] This bargaining protocol is applied to matching models of money in recent studies; see, e.g., Aruoba et al. (2007) and Venkateswaran and Wright (2013). Unlike Nash bargaining, it makes the surplus for an agent increase with his money holdings, implying that agents have no incentive to hide their money holdings. It also preserves the concavity of value functions.

[5] Introduced by Berentsen et al. (2002) into models with indivisible money, lotteries convexify the set of surpluses from trade. Lotteries also mitigate indivisibility. Of course, when money is divisible and $B=\infty$, neutrality is automatic and the exposition is actually simpler. See Zhu (2005) for a sense of approximating divisible money by indivisible money.

[6] The disintegration is introduced by Deviatov and Wallace (2001) to define the individual state by the ratio of the individual money holdings to the stock of money, as in a divisible-money model.

units of transferred money that is fully determined by the transfer policy (C_0, C) and is described in the appendix. Given the value function v_{t+1}, the value function for an agent holding m units of money right prior to the disintegration of money at the end of period t is

$$\tilde{v}_t\left(m\right) = \beta \sum_{m' \leq m} \binom{m}{m'} \left(1-\delta_t\right)^{m'} {\delta_t}^{m-m'} v_{t+1}\left(m'\right), \tag{4}$$

where δ_t is the disintegration probability given $M_t^+ = \sum m \hat{\pi}_t(m)$. Given the value function $\tilde{v}_t$, the trading outcome when a buyer holding m^b meets a seller holding m^s at stage 2 is

$$\left(y_t\left(m^b, m^s\right), \mu_t\left(m^b, m^s\right)\right) = \arg\max_{(y,\mu)} S_t^b\left(y, \mu, m^b\right) \tag{5}$$

subject to

$$\theta S_t^s\left(y, \mu, m^s\right) = \left(1-\theta\right) S_t^b\left(y, \mu, m^b\right), \tag{6}$$

where

$$S_t^b\left(y, \mu, m^b\right) = u\left(y\right) + \sum_d \mu\left(d\right)\left[\tilde{v}_t\left(m^b - d\right) - \tilde{v}_t\left(m^b\right)\right] \tag{7}$$

is the buyer's surplus from trading (y, μ) and

$$S_t^s\left(y, \mu, m^s\right) = -c\left(y\right) + \sum_d \mu\left(d\right)\left[\tilde{v}_t\left(m^s + d\right) - \tilde{v}_t\left(m^s\right)\right] \tag{8}$$

is the seller's. Given the stage-2 meeting outcomes and the distribution $\hat{\pi}_t$, the value for an agent holding m right prior to the stage-2 meetings is

$$\begin{aligned} \hat{v}_t\left(m\right) &= \tilde{v}_t\left(m\right) + 0.5 \sum_{m'} \hat{\pi}_t\left(m'\right)\left[S_t^b\left(y_t\left(m, m'\right), \mu_t\left(m, m'\right), m\right)\right. \\ &+ \left. S_t^s\left(y_t\left(m', m\right), \mu_t\left(m', m\right), m\right)\right]; \end{aligned} \tag{9}$$

the proportion of agents holding m right prior to date-t disintegration of money is

$$\tilde{\pi}_t\left(m\right) = 0.5 \sum_{m^b, m^s} \left[\hat{\lambda}_t^b\left(m, m^b, m^s\right) + \hat{\lambda}_t^s\left(m, m^b, m^s\right)\right] \hat{\pi}_t\left(m^b\right) \hat{\pi}_t\left(m^s\right), \tag{10}$$

where $\hat{\lambda}_t^b(m, m^b, m^s)$ and $\hat{\lambda}_t^s(m, m^b, m^s)$ are the proportion of buyers with m^b and that of sellers with m^s, respectively, ending up with m after those buyers meeting those sellers that are fully determined by the payment lottery $\mu(m^b, m^s)$ and described in the appendix. Finally, the value for an agent holding m at the start of t is

$$v_t(m) = \sum_{m'} \lambda_t(m', m)\, \hat{v}_t(m'); \tag{11}$$

the proportion of agents holding m at the start of $t+1$ is

$$\pi_{t+1}(m) = \sum_{m' \geq m} \binom{m'}{m} (1-\delta_t)^m \, \delta_t^{m'-m} \tilde{\pi}_t(m'). \tag{12}$$

Notice that (3), (10), and (12) determine the law of motion from the distribution π_t to π_{t+1}; (4), (9), and (11) determine the recursive relationship between the value functions v_t and v_{t+1}.

Definition 1 *Given* (π_0, C_0, C)*, a sequence* $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ *is an* equilibrium *if it satisfies* (3)-(12) *all* t*; a pair* (v, π) *is a* steady state *if* $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ *with* $v_t = v$ *and* $\pi_t = \pi$ *all* t *is an equilibrium.*

In an equilibrium $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$, the *aggregate output* at period t is

$$Y_t = 0.5 \sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s)\, y_t\left(m^b, m^s\right), \tag{13}$$

the *average payment* is

$$D_t = \sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s)\, d_t\left(m^b, m^s\right),$$

and the *average price* is

$$P_t = \sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s)\, p_t\left(m^b, m^s\right),$$

where $d_t(m^b, m^s) = \sum_d d\mu_t(d; m^b, m^s)$ and $p_t(m^b, m^s) = d_t(m^b, m^s)/y_t(m^b, m^s)$. We define

$$\varphi_{t+1} = \left(M_t^+/M\right) P_{t+1}/P_t - 1$$

as the *inflation rate*. Given the equilibrium, we can back out the average price at $t+1$ when there were no disintegration at the end of t, which is $(M_t^+/M)P_{t+1}$ (so φ_{t+1} agrees with the change of the average price from t to $t+1$ absent disintegration at

t). Throughout, we remove the time subscript from an object X_t in an equilibrium to represent that object in a steady state.

Our analysis below is quantitative. Except for an exercise in section 5, it mainly involves steady state comparison. Given a set of policy and non-policy parameter values, we compute a steady state (v, π) such that the value function v is strictly increasing and concave—a value function is *concave* if its linear interpolation is concave.[7] The computational procedure follows Jin and Zhu (2019), details of which are given in the appendix. For each set of parameter values experimented, we start from many different initial conditions, but our algorithm always converges to the same steady state. Therefore, we refer to that solved steady state as *the* steady state corresponding to the set of parameter values. Because money is indivisible and the upper bound B on the individual holdings is finite, we use the solved steady state to construct the Jacobian to verify its local stability as in Jin and Zhu (2019).[8]

Most of our analysis reports two statistics for a steady state (v, π): the average expected discount utility or *ex ante (social) welfare*

$$V = \sum_m \pi(m)v(m), \tag{14}$$

and the *indirect risk aversion*

$$\mathit{\Sigma} = \sum_m \pi\left(m\right)\varsigma\left(m\right), \tag{15}$$

where $\varsigma(m)$ is the relative risk aversion at m derived from a smooth approximation of v.

For most of our analysis, we fix non-policy parameter values and vary policy parameter values. Our *benchmark* policy is the no-transfer zero-inflation policy. For non-policy parameter values, we choose a sufficiently large M to mitigate the effects

[7]Zhu (2003) establishes existence of such a steady state when $\theta = 1$. The existence result can be extended to θ sufficiently close to 1, but has not been proved for a general θ.

[8]Molico (2006) numerically solves the divisible-money setup in footnote 5 with $\theta = 1$. We suspect that his algorithm can be extended to at least some range of $\theta < 1$. When we align parameter values with Molico (2006), the results from our indivisible-money setup are almost identical to those reported by him. See Appendix B for a related discussion on computation.

of indivisibility of money, and $M = 30$ serves the purposes well. We choose a sufficiently large B to mitigate the effect of bounding one's nominal wealth; it turns out that $B = 150$ is good enough for most exercises, but we may use a higher value when necessary. We let the annual discount rate be 4%, so that $\beta = 1/(1 + 0.04/F)$ when agents meet F rounds in the decentralized market per year. Unless otherwise stated, the results presented in the paper use $(\sigma, \eta, \omega) = (1, 1, 10^{-4})$ (see (1) and (2)) and $F = 4$. The values of $\sigma = 1$ and η are standard in the literature. The main purpose of ω is to keep the buyer's reservation value (in (7)) well defined; we choose a small value of ω to largely maintain the CRRA property of function u. We discuss different values of $(\sigma, \eta, \omega, F)$ at the end of section 3.

As in Jin and Zhu (2019), we follow Lagos and Wright (2005) in determining the value of the buyer's surplus θ by markup. In a steady state (v, π), let $\kappa(m^b, m^s) = \sum_d \mu(d; m^b, m^s)[v(m^s+d) - v(m^s)]$; we define $\kappa(m^b, m^s)/c(y(m^b, m^s))$ as the (expected) markup in a meeting between a buyer with m^b and a seller with m^s;[9] so the *average markup* at period t is

$$\sum_{m^b, m^s} \hat{\pi}\left(m^b\right) \hat{\pi}\left(m^s\right) \kappa\left(m^b, m^s\right) / c\left(y\left(m^b, m^s\right)\right). \tag{16}$$

We choose 1.39 as the target of the average markup in the benchmark steady state, a target that is at the high end of markup values estimated by empirical studies; this target is suggested by Lagos et al. (2017) and also adopted by Jin and Zhu (2019). Given $(\sigma, \eta, \omega) = (1, 1, 10^{-4})$ and $F = 4$, the average markup reaches 1.39 at $\theta = 0.98$ in the benchmark steady state. We use $\theta = 0.98$ when a policy deviates from the benchmark. An alternative is to identify a different value of θ for which the average markup meets the same target for a different policy. We discuss this alternative at the end of section 3 and more on the different values of θ in section 4.

[9] The seller's surplus can be written as $(\kappa/\iota) \cdot \iota - \psi(\iota)$, where $\iota = c(y(m^b, m^s))$ and $\psi(\iota) = \iota$; that is, the seller exchanges his present utility loss ι due to production with his future utility gain κ due to the monetary payment under the price κ/ι. Treating the seller's surplus as his profit and $\psi(\iota)$ as his total cost, κ/ι is the conventional price-marginal cost markup.

	C_0	C	φ	ΔY	ΔY_π	D	Σ	$Gini$	ΔV
benchmark	0	0	0	0	0	0.21	1.479	0.117	0
regressive	-0.01	0.01	1%	4.94%	4.32%	0.17	1.564	0.183	-1.40%
	-0.02	0.02	2%	11.53%	10.88%	0.15	1.575	0.232	-3.56%
	-0.03	0.03	3%	18.97%	20.66%	0.14	1.562	0.277	-6.48%
progressive	0.3	0	1%	-2.76%	2.38%	0.76	0.783	0.163	0.27%
	0.6	0	2%	-4.59%	5.60%	1.34	0.658	0.205	0.23%
	0.9	0	3%	-6.40%	9.20%	1.92	0.599	0.240	0.15%

Table 1: Steady states under various transfer policies.

3 Real effects of inflation: regressive transfer vs progressive transfer

In this section, we illustrate by examples that regressive transfers have different real effects from progressive transfers. In the examples, we use three inflation targets, 1%, 2%, and 3%. For regressive transfers, we fix $C_0/C = -1$ and set $C_0 = -0.01$, -0.02, and -0.03, corresponding to $\varphi = 1\%$, 2%, and 3%, respectively. For progressive transfers, we fix $C = 0$ and set $C_0 = 0.3$, 0.6, and 0.9, corresponding to $\varphi = 1\%$, 2%, and 3%, respectively. As noted above, the component C_0 is the force that drives the real effects of a transfer. For a regressive transfer, $C > 0$ is necessary to increase the stock of money, and a constant C_0/C keeps the real-effect driving force proportional to φ among regressive transfers; progressive transfers are lump sum transfers, and they are assigned higher values of $|C_0|$ than regressive transfers in order to have real effects comparable in magnitude to those of regressive transfers.[10]

Table 1 reports the main statistics obtained from the benchmark steady state and steady states under the above transfers. Here and below, $\Delta X = X'/X - 1$ represents a relative change of the object X from the zero-inflation benchmark to another steady state, where X and X' are the object's values in the benchmark and the other steady state, respectively. Thus, ΔY is the relative change in aggregate output (ΔY_π is part

[10]Keeping $C_0/C = -1$ among regressive transfers is not crucial, but -1 matches an object that indicates the regressive nature of inflation-financed bonds, i.e., $-g(m)$ in (26); also, see footnote 13 for numbers when the two sorts of transfers share the same values of $|C_0|$.

of ΔY defined below), and ΔV is the relative change in *ex ante* welfare (see (14)). In the table, the Gini of a steady state is the Gini coefficient implied by the steady-state distribution π.

In the rest of this section, we use the indirect risk aversion Σ (see (15)) to understand the real effects of the two sorts of transfers presented in the table. The indirect risk aversion Σ measures the endogenized individual aversion to risk on wealth. An agent is averse to risk on wealth ultimately because it induces a risk on the agent's consumption and production. Here we treat Σ as an indirect indicator of the induced risk on consumption and production experienced by agents, referred to as *the individual risk*; section 4 provides a more detailed analysis of this risk.

A regressive transfer increases the individual risk as it offers more to an agent when he is rich than when he is poor, acting on the benchmark value function as if applying a concave transformation; a progressive transfer does the opposite. The value of Σ is 1.479 at the benchmark, indicating a substantial individual risk which permits a risk-reducing force to reshape the benchmark value function more evidently than a risk-enhancing force. Indeed, Σ moves up to 1.564 for $(C, C_0) = (0.01, -0.01)$ and down to 0.783 for $(C, C_0) = (0, 0.3)$.[11]This can also be seen from Figure 1, which displays steady-state value functions and distributions for $(C, C_0) = (0.01, -0.01)$ $(\varphi = 1\%)$, $(C, C_0) = (0, 0.3)$ $(\varphi = 1\%)$, and $(C, C_0) = (0, 0)$ $(\varphi = 0)$; the figure does not display the value functions over a small neighborhood of zero in which the increments of the value functions for $(C, C_0) = (0.01, -0.01)$ and $(C, C_0) = (0, 0.3)$ are close to 400.

For the wealth distribution, a transfer has an *assignment effect*—it disperses the distribution if $C_0 < 0$ and squeezes it if $C_0 > 0$; it also has a general-equilibrium *expenditure effect*—it disperses the distribution if agents tend to spend more and squeezes it if less. A progressive transfer encourages agents to increase their payments by reducing

[11]By definition, both the change in the value function and the change in the distribution caused by a transfer contribute to the change in the indirect risk aversion. But for each steady state, the standard deviation of $\varsigma(m)$ is no greater than 0.11. On a separate note, a transfer can have a general-equilibrium effect on the value function through its influence on the distribution, but we find that this effect is negligible.

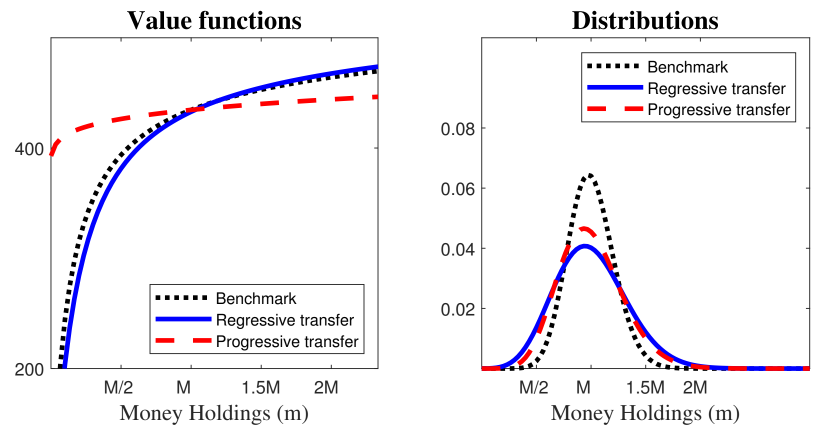

Figure 1: Steady-state value functions and distributions under $(C, C_0) = (0, 0)$ (benchmark), $(C, C_0) = (0.01, -0.01)$ (regressive transfer), and $(C, C_0) = (0, 0.3)$ (progressive transfer), respectively.

the individual risk; a regressive transfer does the opposite. The average payment D is 0.21 at the benchmark, moving up to 0.76 for $(C, C_0) = (0, 0.3)$ and down to 0.17 for $(C, C_0) = (0.01, -0.01)$. A dramatic change in payments due to a progressive transfer may be attributed to a great reduction in the individual risk, and it easily allows the expenditure effect to be the dominant factor. A far more limited change in payments due to a regressive transfer may be because payments are already at a low level at the benchmark, rendering the dominant role to the assignment effect. As such, both transfers in Figure 1 spread the benchmark distribution.

How a transfer reshapes the benchmark value function and distribution helps to explain its effect on aggregate output. By definition,

$$\Delta Y = 0.5 \sum_{m^b, m^s} \left[\hat{\pi}' \left(m^b \right) \hat{\pi}' \left(m^s \right) y' \left(m^b, m^s \right) - \hat{\pi} \left(m^b \right) \hat{\pi} \left(m^s \right) y \left(m^b, m^s \right) \right] / Y.$$

Let the *redistribution effect* of the transfer on aggregate output be defined by

$$\Delta Y_\pi = 0.5 \sum_{m^b, m^s} \left[\hat{\pi}' \left(m^b \right) \hat{\pi}' \left(m^s \right) - \hat{\pi} \left(m^b \right) \hat{\pi} \left(m^s \right) \right] y \left(m^b, m^s \right) / Y,$$

which contributes to ΔY solely by reshaping the distribution (although $\hat{\pi}$ is not the same as π, the two distributions are altered by the transfer similarly). In Table 1, regressive and progressive transfers all have positive and significant redistribution effects. Why? In the benchmark steady state, $y(m^b, m^s+1)+y(m^b, m^s-1)-2y(m^b, m^s)$ and $2y(m^b, m^s)-y(m^b+1, m^s)+y(m^b-1, m^s)$ are positive but the former can significantly exceed the latter (see Figure 4). Now, imagine redistributing wealth between two agents with m so that one has $m+1$ and the other has $m-1$ at stage 1 of a date. This decreases output if both agents become buyers at stage 2 and increases it if both become sellers; but the net effect is positive. Thus, as noted by Jin and Zhu (2019, p. 1337), stretching the distribution reshuffles proportions of meetings with different meeting outputs in a way that leads to a higher aggregate output, if overall incentives to produce are not affected.

Repeated transfers do affect incentives. The part in ΔY complementary to ΔY_π, i.e.,

$$\Delta Y_y = 0.5 \sum_{m^b, m^s} \hat{\pi}'\left(m^b\right)\hat{\pi}'\left(m^s\right)\left[y'\left(m^b, m^s\right) - y\left(m^b, m^s\right)\right]/Y,$$

is the weighted change in incentives, with weights assigned by the distribution $\hat{\pi}'$.[12] Conventional wisdom is that adding more money dilutes incentives to produce. This is the case for a progressive transfer. Indeed, by its way of reshaping the value function, the transfer lowers the incremental values over most money holdings, i.e., lowers $v(m+1)-v(m)$ for most m (how much one unit of money can induce a seller with m to produce depends on $v(m+1)-v(m)$ rather than $v(m+1)$). It is not surprising that the progressive transfer in Figure 1 has ΔY_y dominating ΔY_π; moreover, a larger C_0 undercuts incentives further, leading to a more negative ΔY. A regressive transfer, however, may maintain and even further enhance incentives because its way of reshaping the value function largely maintains and even raises the incremental values of money; with the ratio of C_0 to C being fixed, inflation and ΔY both increase as the risk-enhancing force $|C_0|$ increases.

How each transfer reshapes the benchmark value function and distribution is suffi-

[12]Recall that the change in $\hat{\pi}$ affects the value function $\tilde{v}$ by affecting the disintegration probability. So strictly speaking, the changes in v and $\hat{\pi}$ contribute to the change in the meeting output. However, the change in v is the dominant factor because the change in $\hat{\pi}$ mainly shifts the entire function $\tilde{v}$ down, while the change in v affects the incremental values of $\tilde{v}$.

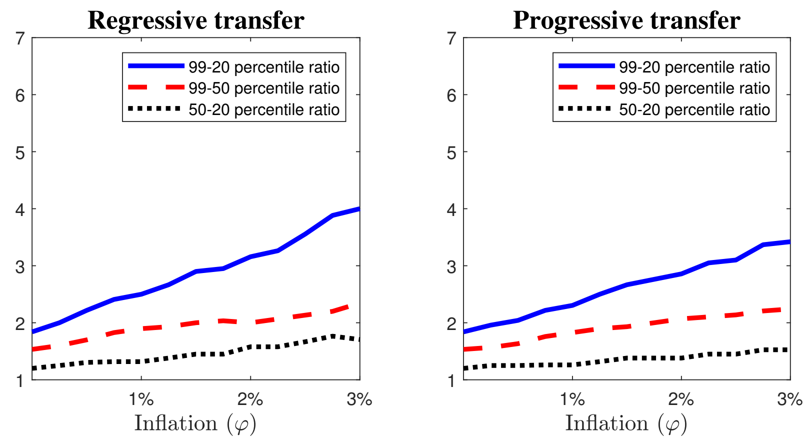

Figure 2: Selected percentile ratios of the wealth distribution under different inflation rates generated by regressive or progressive transfers.

cient to explain the change in *ex ante* welfare. Alternatively, one may think that the substantial individual risk at the benchmark is not desirable so that a regressive transfer reduces welfare, while a progressive transfer can improve it (at least for inflation in some range).

The Gini values tell that the spread of wealth is responsive to inflation. Figure 2 presents three different ratios of percentiles of the wealth distribution, where "*a*-*b* percentile ratio" is the ratio between wealth levels at the *a*th and the *b*th percentiles. The pattern in Figure 2 fits well with a key feature observed from the data: wealth becomes more concentrated at the very top.[13]

The main findings presented so far are summarized as follows.

Result 1 *A regressive transfer induces a positive redistribution effect on output and maintains overall incentives to produce. A progressive transfer may induce a positive redistribution effect but it undercuts overall incentives. Only a regressive transfer can increase output significantly. A regressive transfer reduces* ex ante *welfare while a progressive transfer may improve it. Both sorts of transfers make wealth more concentrated at the very top.*

[13]When the values of C_0 in progressive transfers are 0.01, 0.02, and 0.03, the corresponding values of $(\Delta Y, \Delta V, \text{Gini})$ are $(-0.04\%, 0.05\%, 0.119)$, $(-0.08\%, 0.09, 0.122)$, and $(-0.11\%, 0.12\%, 0.123)$, respectively.

The Table-1 exercise uses the baseline values of the meeting frequency F, the risk aversion coefficient σ in u, the labor supply elasticity η in c, and the constant term ω in u. We run experiments by varying one parameter at a time for the robustness check. When F increases from 1 to 365, Σ varies narrowly between 1.50 and 1.47. When ω increases from 10^{-6} to 10^{-2}, Σ decreases from 1.66 to 1.19. When η increases from 0.25 to 4.0, Σ decreases from 1.84 to 1.28. When σ increases from 0.5 to 1.5, Σ increases from 0.81 to 1.90. The changes in Σ are intuitive: one is more averse to risk on wealth if he is more averse to risk on consumption, more averse to risk on production, or has a larger ω to self-insure; his aversion should not have much to do with F. While details vary, the patterns in Table 1 remain valid in these experiments; Table 4 in Appendix C reports the statistics for selected parameter values. In particular, we find no counterexamples to Result 1 except for a sufficiently small η. After all, our explanation above for these results relies on two properties of the model: (i) there is a substantial individual risk at the benchmark; and (ii) the individual risk can be significantly reduced by a progressive transfer. The exception due to a small η is that a progressive transfer squeezes the distribution (its assignment effect becomes the dominant factor). To reconcile the exception with the above explanation, note that when buyers spend more, sellers should produce more in equilibrium. Thus, though leading to a larger Σ, a smaller η may impose a more severe constraint on more production and result in a smaller expenditure effect. Of course, when the distribution is squeezed, wealth may be less concentrated at the top.[14]

In the Table-1 exercise, we also fix the buyer's surplus weight θ at 0.98. Alternatively, we may identify a different value of θ by which the average markup meets the markup target for a different policy. As it turns out, the different value of θ is quite close to 0.98 (e.g., if $\varphi = 3\%$ then it is 0.988 and 0.982 for the regressive and progressive transfers, respectively); but, as anticipated, a small change in θ does not

[14]Even when η is small, a regressive transfer still maintains a strong redistribution effect. One may relate this to the finding in Jin and Zhu (2019): a one-shot regressive transfer cannot sustain a strong and positive output effect when η becomes small because it exerts a much weakened dispersion force on the distribution.

affect the numbers in Table 1 much. The effects due to a large change in θ are shown in section 4.

4 Role of decentralized trade

To illustrate how decentralized trade contributes to the findings in section 3, now we replace decentralized trade in the basic model with centralized trade as follows. At stage 2 of each date t, agents trade in a centralized market where they take the price of money ϕ_t as given. A trading outcome for an agent carrying m into the market is (y, μ): if the agent is a buyer, he receives y units of goods from the market and pays to the market $d \in \{0, \ldots, m\}$ units of money with probability $\mu(d)$; if he is a seller, he surrenders y units of goods to the market and receives from the market $d \in \{0, \ldots, B-m\}$ units of money with probability $\mu(d)$; and the mean of the distribution μ is y/ϕ_t. All other aspects of the basic model are unchanged.

Given the constraint of ϕ_t imposed on trading outcomes, equilibrium conditions at period t are again described by the value function v_{t+1} and the distribution π_t. As above, π_t and (C, C_0) fully determine the distribution $\hat{\pi}_t$; for an agent carrying m into the market, the surplus $S_t^b(y, \mu, m)$ from a trading outcome (y, μ) when he is a buyer and the surplus $S_t^s(y, \mu, m)$ when he is a seller are fully determined by v_{t+1} and $\hat{\pi}_t$. The agent's trading outcome is

$$\begin{aligned} \left(y_t^a(m), \mu_t^a(.; m)\right) &= \arg\max_{(y,\mu)} S_t^a(y, \mu, m), \\ \text{s.t. } y_t^a(m) &= \phi_t \sum_d d\mu_t^a(d; m), a \in \{b, s\}. \end{aligned} \tag{17}$$

Market clearing requires

$$\sum_m \hat{\pi}_t(m) \sum_d d\mu_t^b(d; m) = \sum_m \hat{\pi}_t(m) \sum_d d\mu_t^s(d; m). \tag{18}$$

Given (π_0, C_0, C), a sequence $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ is an *equilibrium* under centralized trade if it satisfies the recursive relationship between the value functions v_t and v_{t+1}, the law of motion from the distribution π_t to π_{t+1}, and the market clearing condition (18),

	C_0	C	φ	ΔY	ΔY_π	D	Σ	$Gini$	ΔV
benchmark	0	0	0	0	0	4.98	0.382	0.267	0
regressive	-0.01	0.01	1%	0.17%	-0.003%	4.93	0.381	0.268	0.003%
	-0.02	0.02	2%	0.33%	-0.002%	4.87	0.379	0.268	0.009%
	-0.03	0.03	3%	0.49%	0.001%	4.82	0.378	0.268	0.015%
progressive	0.3	0	1%	-3.29%	-0.12%	6.71	0.428	0.285	-0.32%
	0.6	0	2%	-5.86%	-0.23%	8.15	0.440	0.303	-0.56%
	0.9	0	3%	-8.03%	-0.38%	9.56	0.421	0.323	-0.77%

Table 2: Steady states under various transfer policies: centralized trade.

all t; a tuple (v, π, ϕ) is a *steady state* if $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ with $(v_t, \pi_{t+1}, \phi_t) = (v, \pi, \phi)$ all t is an equilibrium. Details of equilibrium conditions are given in the appendix. Now in an equilibrium, the aggregate output at period t is $Y_t = 0.5 \sum_m \hat{\pi}_t(m)\, y_t^s(m)$ and the average payment is $D_t = \sum_m \hat{\pi}_t(m) \sum_d d\mu_t^b(d; m)$. Table 2 reports the (steady-state) statistics under centralized trade for the same values of (C, C_0) used in Table 1. Again, we appeal to the individual risk to understand the real effects of the two sorts of transfers presented in the table.

The value of Σ is 0.382 at the benchmark, indicating a much mild individual risk. The mild individual risk means that the benchmark value function is much flatter than its counterpart in Figure 1, constraining the room for a risk-changing force to reshape the benchmark value function. This can be seen in Figure 3, which displays the steady-state value functions and distributions when $(C, C_0) = (0.01, -0.01)$, $(C, C_0) = (0, 0.3)$, and $(C, C_0) = (0, 0)$ under centralized trade. The value function when $(C, C_0) = (0.01, -0.01)$ closely follows the benchmark value function. Left to the mean holdings M, the value function when $(C, C_0) = (0, 0.3)$ moves up from the benchmark by a more limited degree than its counterpart in Figure 1, indicating that the insurance benefit to the poor agents of the transfer is much weakened. One may conclude that a regressive transfer at least maintains the overall incentive to produce while a progressive transfer does not.[15]

With the mild individual risk at the benchmark, agents tend to spend much more

[15] One may also note $\Sigma = 0.381$ when $(C, C_0) = (0.01, -0.01)$ and $\Sigma = 0.428$ when $(C, C_0) = (0, 0.3)$, telling that Σ is an approximate reference for the global curvature of a function.

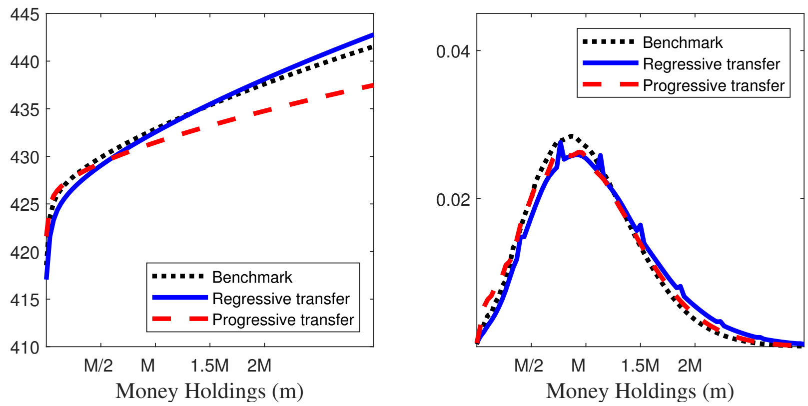

Figure 3: Steady-state value functions and distributions under $(C, C_0) = (0, 0)$ (benchmark), $(C, C_0) = (0.01, -0.01)$ (regressive transfer), and $(C, C_0) = (0, 0.3)$ (progressive transfer), respectively: centralized trade.

$(D = 4.98)$. This explains why distributions in Figure 3 are more dispersed than their counterparts in Figure 1. Given that the average payments are already high, there is little room for the progressive transfer to generate the expenditure effect on the distribution that may dominate the assignment effect. However, there may be some room for the regressive transfer to generate the expenditure effect that may somewhat offset the assignment effect. This explains why in Figure 3, the distribution is squeezed (relative to the benchmark) by the progressive transfer, shifted to the right by the regressive transfer, and is reshaped much less by either transfer than is its counterpart in Figure 1. One may conclude that a regressive transfer has an ambiguous and rather small redistribution effect, while a progressive transfer has a negative and not large redistribution effect; now $\Delta Y = 0.5 \sum_m [\hat{\pi}'(m) y^{s\prime}(m) - \hat{\pi}(m) y^s(m)]/Y$ and the redistribution effect $\Delta Y_\pi = 0.5 \sum_m [\hat{\pi}'(m) - \hat{\pi}(m)] \, y^s(m) / Y$.

In summary, under centralized trade, the positive output-inflation correlation for regressive transfers is weakened because the redistribution effects are weakened; the negative output-inflation correlation for progressive transfers is strengthened because the redistribution effects become negative. And, because both sorts of transfers have a weakened influence on the distribution, the Gini is not much responsive to inflation. Moreover, given the mild individual risk at the benchmark, a progressive transfer re-

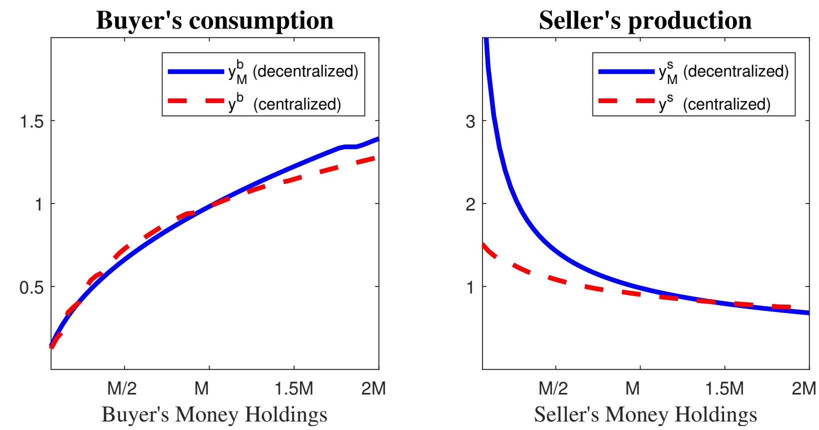

Figure 4: Present consumption and production as a function of m.

duces *ex ante* welfare because its insuring benefit is limited and actually dominated by the loss in aggregate output; a regressive transfer may be welfare-improving because of the gain in aggregate output.

Result 2 *Under centralized trade, there is a much mild individual risk at the benchmark, which is consistent with the real effects of each transfer that differ from those under decentralized trade.*

We continue to explore what may be the channel for decentralized trade to amplify the individual risk at the benchmark. The individual risk measured by Σ refers to the individual risk on consumption and production, induced by a risk on wealth of an agent. To analyze this risk, recall that $m \to y^b(m)$ and $m \to y^s(m)$ (see (17)) define the present consumption and the present production under centralized trade as functions of an individual agent's money holdings, respectively, and that $m \to y^b_n(m) \equiv y(m,n)$ and $m \to y^s_n(m) \equiv y(n,m)$ (see (5)) define those functions under decentralized trade conditional on the meeting partner's holdings n. Let $w^b(m) = u(y^b(m))$ and $w^s(m) = -c(y^s(m))$ under centralized trade; let $w^a(m) = \sum_n \pi(n) w^a_n(m)$ under decentralized trade, where $a \in \{b,s\}$, $w^b_n(m) = u(y^b_n(m))$, and $w^s_n(m) = -c(y^s_n(m))$. Let $\gamma(m,w)$ be the relative risk aversion of $w = w^a$ at m (based on a smooth approximation of w) and let $\Gamma(w)=\sum_m \pi(m)\gamma(m,w)$. Then $\gamma(m,w^a)$ measures how much an agent holding m

is averse to the risk on the *present* consumption (if $a = b$) or production (if $a = s$) induced by a small shock to his wealth and, then, $\Gamma(w^a)$ tells on average how much an agent is averse to that risk. At the benchmark, $\Gamma(w^b) = 1.25$ and $\Gamma(w^s) = 1.35$ under centralized trade; $\Gamma(w^b) = 1.11$ and $\Gamma(w^s) = 2.53$ under decentralized trade and $\left|\Gamma(w^b_n) - 1.11\right|$ and $|\Gamma(w^s_n) - 2.53|$ are smaller than 0.001 for n between $0.5M$ and $1.5M$ (the mass of agents with n in this range represents 99% of the population).

For an alternative interpretation of $\Gamma(w^a)$, think of an agent behind the veil of ignorance—he has no money but knows that the economy is in the benchmark steady state, he is about to receive money by the distribution π before his current type is realized, and his wealth is to be hit by a small shock after he receives the money. Suppose the current calendar date is zero, and denote by $\varpi_t(m)$ the probability that he holds m units of money before date-t trade. Focus on decentralized trade. The transition matrix between ϖ_t and ϖ_{t+1} is fully determined by π and $\mu(.,.)$ (see (5)). In the absence of the date-0 shock, $\varpi_0 = \pi$, implying that $\varpi_t = \pi$ all $t > 0$; therefore, behind the veil of ignorance, the agent perceives that at date $t \geq 0$ and with probability $\pi(m)$, he consumes $y^b_n(m)$ and produces $y^s_n(m)$ (conditional on the meeting partner's holdings n). Because of the date-0 shock, $\varpi_0 \neq \pi$. As π has full support (found by computation), the transition matrix has the properties that ϖ_t converges to π and that ϖ_t always stays within any given distance to π if the date-0 shock is sufficiently small. Numerical experiments indicate that in general the movement from ϖ_t to π is smooth: if $\varpi_t(m) \gtreqless \pi(m)$ then $\varpi_t(m) \gtreqless \varpi_{t+1}(m) \gtreqless \pi(m)$. The smooth movement allows ϖ_t at $t \geq 1$ to be approximated by an outcome of the following hypothetical scenario: with probability $\pi(m)$, the agent holds m at the start of t and then there is a small shock to his wealth. The agent, therefore, perceives behind the veil of ignorance that, as his money holdings stochastically vary around m, his consumption varies around $y^b_n(m)$ and production around $y^s_n(m)$. For centralized trade, the analogue holds. So $\Gamma(w^a)$ measures how much the agent behind the veil of ignorance is averse to the risk on consumption (if $a = b$) or production (if $a = s$) *in each future date* induced by the date-0 shock and, thus, it is treated as a direct indicator of the individual risk on consumption or production (not only the present consumption or production).

But why does decentralized trade amplify the individual risk mainly by way of the individual risk on production? Figure 4 displays functions y^b, y^b_M, y^s, and y^s_M (the shapes of y^b_M and y^s_M are representative of those of y^b_n and y^s_n for $n \neq M$). Some intuition may be helpful to understand the shapes of these functions. Trade being centralized or not, an agent accumulates wealth to partially insure himself against the fundamental risk of the model, i.e., the idiosyncratic shock that determines his type at each period; specifically, he produces as a seller to insure his consumption as a buyer. Now think of that a hypothetical shock causes the agent to win or lose one unit of money by equal chance at stage 1. Losing money at stage 1, the agent adjusts his wealth upward with respect to the certainty case, i.e., the case without the hypothetical shock, at stage 2 by spending less as a buyer and earning more as a seller than in the certainty case; winning money, he adjusts downward by doing the opposite. For insurance purposes, the agent would be urgent to let the adjusting-up degree exceed the adjusting-down degree. This urgency translates into convexity of y^s and y^s_n, and concavity of y^b and y^b_n. In the absence of competition, the agent's urgency to increase earnings after losing may be exploited by his meeting partner, making y^s_n more convex than y^s; likewise, his urgency to reduce spending after losing may give himself some advantage, making y^b_n less concave than y^b.[16]

So far, we set the buyer's weight θ of surplus sharing at 0.98. There ought to be some dependence of Σ on θ (if $\theta = 0$ then $\Sigma = \Gamma(w^b) = \Gamma(w^s) = 0$, as money becomes valueless). Now we vary θ from 1 to 0.35 (our algorithm does not converge after θ falls somewhere below 0.35). Figure 5 displays the corresponding benchmark steady-state values of $\Gamma(w^b)$, $\Gamma(w^s)$, and Σ together with the output changes due to the transfers with $(C, C_0) = (0.01, -0.01)$ and $(C, C_0) = (0, 0.3)$; the figure also draws a cutoff value $\hat{\theta} = 0.575$ such that *ex ante* welfare can be improved by some progressive transfer

[16]Functions y^b, y^b_n, y^s, and y^s_n are shaped by many general-equilibrium forces. As such, we cannot prove the intuition even though it may be helpful. In fact, as is clear in Figure 4, y^b is not globally concave and y^s is not globally convex.

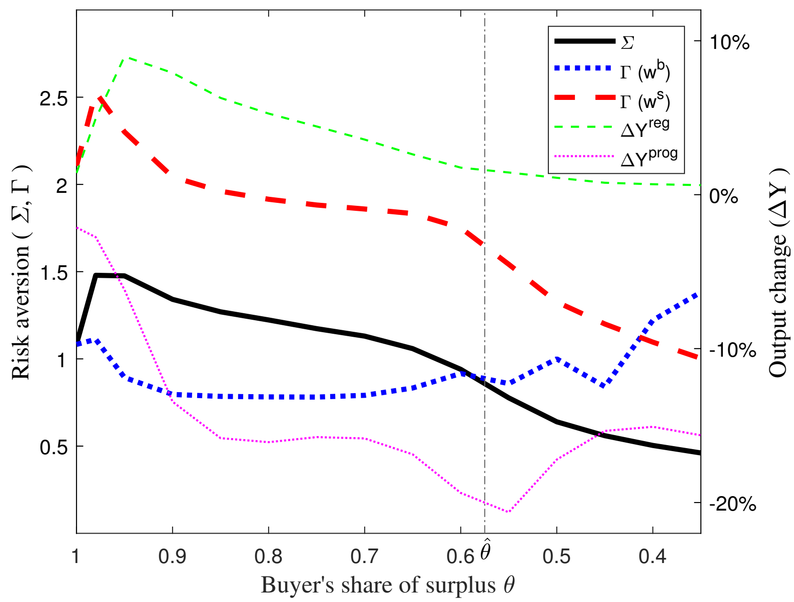

Figure 5: Left axis: $\mathit{\Sigma}$, $\Gamma(w^b)$, and $\Gamma(w^s)$ at benchmark under different θ; right axis: output changes (ΔY) due to 1% regressive and 1% progressive transfers, under different θ.

(regressive transfer, resp.) when $\theta > \hat{\theta}$ ($\theta < \hat{\theta}$, resp.). In the figure, $\mathit{\Sigma}$ moves up as θ moves down from unity over a small range; outside this range, $\mathit{\Sigma}$ moves down with θ. Given the trend of $\mathit{\Sigma}$, the trend of the output change of each transfer and the presence of the cutoff point $\hat{\theta}$ are consistent with our explanation in section 3.

In Figure 5, $\Gamma(w^s)$ closely traces $\mathit{\Sigma}$. The trend of $\Gamma(w^s)$ conforms well with the abovementioned intuition: the agent's urgency to adjust earnings as a seller is more easily exploited when his meeting partner has more bargaining power. We attribute the exception over a small range of θ close to unity to the fact that there is no effective two-sided bargaining at $\theta = 1$, allowing two-sided bargaining alone to be a significant factor influencing the individual risks for both sides as θ slightly departs from unity. In the figure, $\Gamma(w^b)$ is more flattened than is $\mathit{\Sigma}$ over a wide range of θ, but there is an apparent upward trend of $\Gamma(w^b)$ after θ decreases further from 0.5. Over the entire range, the individual risk on production contributes much more to the individual risk than does the individual risk on consumption. As anticipated, the average benchmark markup is decreasing in θ: it is 1 when $\theta = 1$, moves up to 2.11 when $\theta = 0.95$, and further to 7.14 when $\theta = 0.7$.

Result 3 *For a range of θ consistent with a very wide range of markup values starting from unity, decentralized trade amplifies the individual risk through the labor income earning channel—bilateral bargaining amplifies the individual risk on production, and Result 1 remains valid.*

We conduct a robustness check for the impact of θ on the individual risks, output change, and welfare change by varying σ and η. The basic patterns in Figure 5 are maintained. Table 5 in Appendix C reports the statistics for selected parameter values.

To conclude this section, we provide a brief comparison with Molico (2006), who studies the same model with divisible money for $\theta = 1$ (see footnote 8). When θ approaches 1 from 0.98, we find that a progressive transfer with a sufficiently small C_0 under baseline $(F, \omega, \sigma, \eta)$ slightly increases output and *ex ante* welfare. This is consistent with what Molico (2006) reports. Molico (2006) also reports that a progressive transfer squeezes the distribution with sufficient small C_0. This is consistent with our finding for small η. Actually, Molico's disutility function quickly becomes much more convex than ours when meeting output moves away from the static efficient level y^* ($u'(y^*) = c'(y^*)$) to the right, which, as noted in Jin and Zhu (2019, section 6), prevents a strong and positive redistribution effect from each sort of transfer.

5 The model with nominal bonds

Here we add government nominal bonds to the section-2 model for two related purposes. First, it demonstrates that conduction of regressive transfers does not require the government to monitor the individual's money holdings. In fact, the regressive nature of financing nominal bonds by inflation has been noted by Wallace (2014). Second, inflation in reality may be hybrid in that it is neither purely regressive nor purely progressive, and we intend to extend our study to such a policy. With bonds, the government can run a class of hybrid policies according to the individual's bond holdings.

Now at stage 1 of period t, the government issues nominal bonds on a competitive market; each unit of bonds automatically turns into one unit of money at the end of period t. Each agent chooses a probability measure $\hat{\mu}$ (a lottery) defined on the set $\Xi = \{\zeta = (\zeta_1, \zeta_2) \in \mathbb{Z}_+ \times \mathbb{Z}_+ : 1 \leq \zeta_1 + \zeta_2 \leq B\}$ that satisfies

$$\sum_{\zeta=(\zeta_1,\zeta_2)} \hat{\mu}(\zeta) \cdot \left[\zeta_1 + \zeta_2 (1 + i_t)^{-1}\right] \leq m, \tag{19}$$

where m is the amount of money carried by the agent into the market, i_t is the nominal interest rate at t (i.e., $(1+i_t)^{-1}$ is the price of bonds) set by the government who stands to meet any demand on bonds, and $\hat{\mu}(\zeta)$ is the probability that the agent leaves the bond market with the portfolio $\zeta = (\zeta_1, \zeta_2)$ consisting of ζ_1 units of money and ζ_2 units of bonds. After the bond market is closed, the government transfers money to agents in the form of lotteries as in section 2. What is new here is that how much an agent receives depends on his bond holdings instead of his money holdings. A transfer policy is represented by some $K \geq 0$: if the lottery chosen by an agent on the bond market is realized as some $\zeta = (\zeta_1, \zeta_2)$, then the transfer policy assigns to the agent a lottery $\tilde{\mu}(.;\zeta)$ with a mean equal to $\min\{K(1+\zeta_2)^{-1}, B - \zeta_1 - \zeta_2\}$ and the minimal variance. A transfer policy is *active* if $K > 0$ and *inactive* if $K = 0$.

At stage 2, agents are matched in pairs as in the section-2 model. In each meeting, each agent can observe his meeting partner's portfolio, but bonds are illiquid and money is the unique payment method. After the meeting, bonds mature and the money stock is

$$M_t^+ = M_t + L_t \left[1 - (1 + i_t)^{-1}\right] + \tilde{K}_t,$$

where L_t is the stock of bonds and $\tilde{K}_t$ is the sum of the transfer. The interest payments $L_t[1-(1+i_t)^{-1}]$ are financed by inflation. Analogous to the section-2 model, each unit of money disintegrates with the probability that restores the nominal stock back to $M_t = M$ at the end of t.

The equilibrium conditions are described by a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$, where v_t and π_t are the same as in the section-2 model and $\hat{\pi}_t$ is the distribution of portfolios right before pairwise meetings at period t. (We need $\hat{\pi}_t$ as a construct independent from (v_t, π_t) to deal with that the individual portfolio choice is endogenous.) Now the value for an agent holding the portfolio ζ at the end of pairwise meetings is

$$\tilde{v}_t(\zeta) = \beta \sum_{m' \leq \zeta_1+\zeta_2} \binom{\zeta_1+\zeta_2}{m'} (1-\delta_t)^{m'} \delta_t^{\zeta_1+\zeta_2-m'} v_{t+1}(m'), \tag{20}$$

where δ_t is the disintegration probability given by $M_t^+ = \sum_\zeta (\zeta_1+\zeta_2)\hat{\pi}(\zeta)$; the trading outcome $(y_t(\zeta^b, \zeta^s), \mu_t(\zeta^b, \zeta^s))$ when a buyer holding ζ^b meets a seller holding ζ^s at stage 2 is determined by (5) with ζ^b substituting for m^b and ζ^s substituting for m^s; the value for an agent holding ζ right before the stage-2 meetings is

$$\hat{v}_t(\zeta) = \tilde{v}_t(\zeta) + 0.5 \sum_{\zeta'} \hat{\pi}_t(\zeta') \left[S_t^b(y_t(\zeta,\zeta'), \mu_t(\zeta,\zeta'), \zeta) + S_t^s(y_t(\zeta',\zeta), \mu_t(\zeta',\zeta), \zeta) \right]; \tag{21}$$

and the proportion of agents holding ζ right before date-t disintegration of money is

$$\tilde{\pi}_t(\zeta) = 0.5 \sum_{\zeta'} \left[\hat{\lambda}_t^b(\zeta, \zeta^b, \zeta^s) + \hat{\lambda}_t^s(\zeta, \zeta^b, \zeta^s) \right] \hat{\pi}_t(\zeta^b) \hat{\pi}_t(\zeta^s), \tag{22}$$

where $\hat{\lambda}_t^b(\zeta, \zeta^b, \zeta^s)$ and $\hat{\lambda}_t^s(\zeta, \zeta^b, \zeta^s)$ are analogous to $\hat{\lambda}_t^b(m, m^b, m^s)$ and $\hat{\lambda}_t^s(m, m^b, m^s)$ in (10). The portfolio choice problem for an agent holding m can be expressed as

$$v_t(m) = \max_{\hat{\mu}} \sum_{\zeta=(\zeta_1,\zeta_2)} \hat{\mu}(\zeta) \left[\sum_z \tilde{\mu}(z;\zeta) \hat{v}_t(\zeta_1+z, \zeta_2) \right]. \tag{23}$$

subject to (19). Let $\hat{\mu}_t(.;m)$ be the $\hat{\mu}$ that solves the problem (23). Then the proportion of agents holding ζ prior to pairwise meetings is

$$\hat{\pi}_t(\zeta) = \sum_{\zeta'} \left[\tilde{\mu}(\zeta_1 - \zeta_1', \zeta') \sum_m \hat{\mu}_t(\zeta'; m) \pi_t(m) \right]. \tag{24}$$

The proportion of agents holding m at the start of $t+1$ is

$$\pi_{t+1}(m) = \sum_{\zeta_1+\zeta_2 \geq m} \binom{\zeta_1+\zeta_2}{m} (1-\delta_t)^m \delta_t^{\zeta_1+\zeta_2-m} \tilde{\pi}_t(\zeta). \tag{25}$$

Definition 2 *Given π_0, K, and $\{i_t\}_{t=0}^{\infty}$, a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ is an* equilibrium

i (annual)	1%	2%	4%	8%	10%
φ (annual)	0.97%	1.93%	3.87%	7.73%	9.67%
ΔY	1.00%	2.13%	4.75%	11.06%	14.53%
$i-\varphi$	0.03%	0.07%	0.13%	0.27%	0.33%
Gini	0.135	0.152	0.181	0.230	0.250
ΔV	−0.27%	−0.58%	−1.34%	−3.39%	−4.68%

Table 3: Real effects of inflation-financed bonds.

if it satisfies (20)-(25) *all* t. *If* $i_t = i$ *all* t, *a tuple* $(v, \hat{\pi}, \pi)$ *is a* steady state *if* $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ *with* $(v_t, \hat{\pi}_t, \pi_{t+1}) = (v, \hat{\pi}, \pi)$ *all* t *is an equilibrium.*

The quantitative analysis here follows the same procedure and adopts the same parameter values as in section 2. The benchmark policy is the one with no transfer and zero interest.

Inactive transfer policy ($K = 0$)

With $K = 0$, inflation is all driven by interest payments, and thus inflation increases as the nominal interest rate increases. Table 3 displays inflation, the change in output (with respect to the benchmark), the real interest rate, the Gini, and the change in *ex ante* welfare for each of five selected values of the nominal interest rate. In the table, the output-inflation correlation resembles that in Table 1 for regressive transfers (note that a period is a quarter, so annual nominal interest and annual inflation are $4i$ and 4φ, respectively); moreover, as the regressive transfers in Table 1, inflation reduces *ex ante* welfare and the wealth Ginis are quite responsive to inflation.

The statistics in the two tables are similar because financing nominal bonds by inflation is regressive. To see this, notice that the expected interest payments for an agent who enters the bond market with m units of money are

$$[m - g(m)]\, i = -g(m)\, i + im. \tag{26}$$

where $g(m)$ is the amount of money implied by the agent's portfolio choice. If $g(m)/m$ decreases in m, bonds serve as a regressive transfer. While $g(m)$ (weakly) increases in

m, it has a narrow range. Recall that the average spending at the benchmark is far less than unity. So when i is positive, one chooses to carry one unit of money into stage 2 unless he has no money ($m = 0$) or is very rich;[17] as it turns out, more than 99% of agents choose to carry one. Applying $g(m) = 1$ to (26), raising i is equivalent to raising C and keeping $C_0/C = -1$ in the section-2 model.

Table 3 also shows a violation of the Fisher equation; that is, inflation rises less than one-for-one with the nominal interest rate. Given $K = 0$, the newly injected money φM within a period is all used to finance the interest payments $iL/(1+i)$, i.e., $(\varphi - i)M = i[L/(1+i) - M]$. If inflation rises on a one-for-one basis, then $M = L/(1+i)$, meaning that all agents should spend all money on bonds to maintain the Fisher equation, which is clearly impossible. This violation of the Fisher equation can be an equilibrium in our model because no equilibrium condition in the model forces the real interest rate to be a constant.[18]

Welfare-neutral active policy

When $i > 0$, there can be many hybrid policies resulting from different values of K. As a reference, we choose a value of K for a given $i > 0$, denoted $K(i)$, such that the transfer is just progressive enough to offset the regressive nature of bonds. i.e., the corresponding steady state delivers the same *ex ante* welfare as the benchmark steady state. The pair $(i, K(i))$ constitutes a *welfare-neutral active policy.* Figure 6a displays the output-inflation correlation when i rises in the welfare-neutral active policies. Along the path of inflation in Figure 6, Ginis for wealth range from 0.147 to 0.233, similar to those in Table 3. The correlation pattern fits well with the empirical finding of Bullard and Keating (1995); that is, inflation mildly expands output over a

[17] When the annual nominal interest rate is 2%, one carries 2 if $134 \geq m > 76$ and 3 if $m > 134$. Although the integer property of $g(m)$ is a consequence of indivisibility, that $g(m)/m$ decreases in m should hold with divisible money.

[18] For comparison, consider the Lagos-Wright model (2005) with government bonds. The real interest rate there must be equal to the inverse of the discount factor. If inflation is entirely driven by financing bonds, there is no monetary (steady state) equilibrium when the nominal interest rate is positive but sufficiently close to zero.

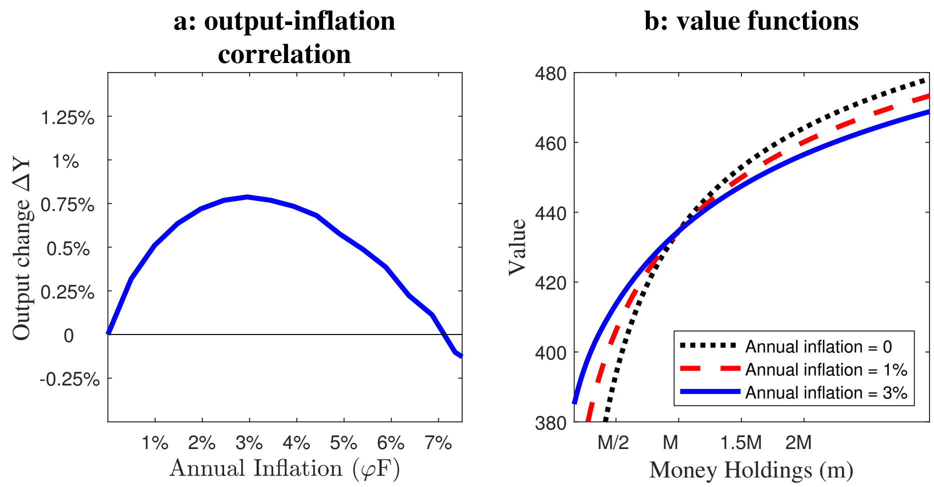

Figure 6: Output-inflation correlations and value functions associated with welfare-neutral active policies.

limited range and the expanding effect gradually phases out beyond this range.

To understand this pattern, we display in Figure 6b value functions with annual inflation at 0, 1%, and 3%. We make the following observation: in partial equilibrium, a rise in i strengthens the regressive feature of the policy but, in general equilibrium, the rise in K to maintain *ex ante* welfare at the benchmark level effectively leads the entire policy to perform as a more progressive policy than the one with lower i. So when i becomes larger, the value function becomes more flattened and, consistent with our analysis in section 3, the distribution becomes more dispersed. In short, the progressiveness of a welfare-neutral policy increases in i. Consequently, when i is small, a low degree of progressiveness allows the redistribution effect ΔY_π to dominate the incentive effect ΔY_y, leaving some room for the regressive aspect of the policy to increase output; this dominance is reversed by a high degree of progressiveness as i grows.

Individual responses to potential rises in inflation

In a world where people already have different levels of wealth, how may one respond to a potential rise in inflation according to wealth status? In particular, is there a basis for the whole of society to consider a welfare-neutral active policy? To shed some light on this issue, we take our analysis beyond steady-state comparisons. To describe our approach, let the economy already reach some steady state $(v, \hat{\pi}, \pi)$ under some prevailing policy. Suppose that a policy alternative to the prevailing policy is implemented at the start of the present date, and let $(v', \hat{\pi}', \pi')$ be the steady state corresponding to the alternative policy. Reset the calendar time so that the present date is date 0 and let $\{v'_t, \hat{\pi}'_t, \pi'_{t+1}\}_{t=0}^{\infty}$ denote the transitional equilibrium connecting the two steady states, i.e., it starts from the initial distribution $\pi'_0 = \pi$ and converges to $(v', \hat{\pi}', \pi')$ as t goes to ∞. For an agent holding m units of nominal wealth at the start of date 0, $v(m)$ is his (life-time) welfare measured at date 0 if there is no policy change and $v'_0(m)$ is his welfare if the alternative policy is adopted. We measure the agent's response to the potential policy change by

$$\rho\,(m) \equiv v'_0\,(m)\,/v\,(m) - 1, \tag{27}$$

the *ex-post* welfare change for the agent if the alternative policy is adopted.

We run an exercise with $(v, \hat{\pi}, \pi)$ being the benchmark steady state and with three alternative policies. The first policy is regressive. The second is the welfare-neutral active policy which has the same i as the regressive policy. The third policy is progressive: a lump-sum policy that yields the highest ex ante welfare among all lump sum policies. The values of (K, i) for these policies are $(0, 3\%/4)$, $(0.089, 3\%/4)$, and $(0.174, 0)$, respectively. Figure 7 displays three ρ functions. The figure has three important patterns representative of other policy parameter values. First, no inflation policy wins a majority support. Second, agents in the middle of π are not sensitive to which policy is adopted; moving away from the middle, agents become increasingly sensitive; but the change in individual sensitivity is more pronounced as moving to the poor end. Third, poor agents disfavor a regressive policy much more than rich agents favor the policy; and poor agents favor a progressive or a welfare-neutral active policy more than rich agents disfavoring it.

Two lessons emerge. First, it may be too simple to only count the number of people

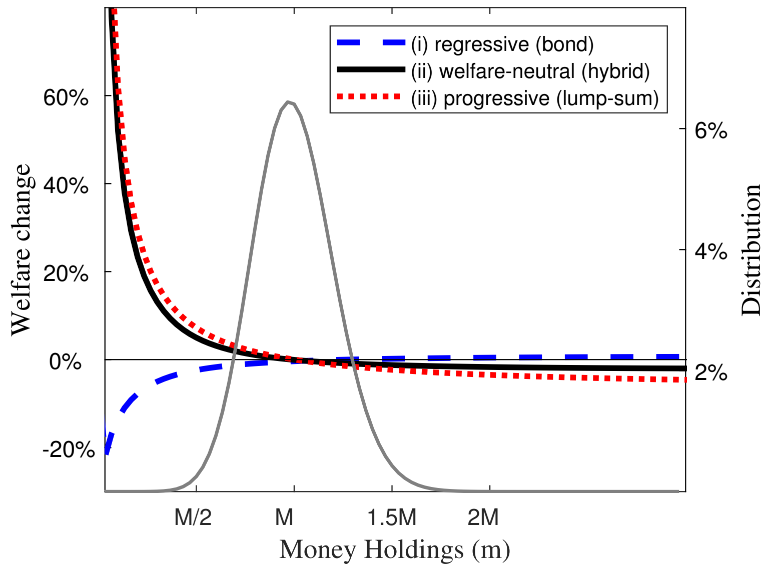

Figure 7: Changes in individual welfare ($\rho(m)$).

who favor a policy while ignoring the degree by which a certain group of people favor or disfavor the policy. In particular, the demand for insurance by the poor in society may be a dominant factor in social choice, even though this demand is disfavored by the rich. Second, a welfare-neutral policy may be attractive because it better balances the demands from the two sides.

Result 4 *The two sides of society may respond much differently to different inflation policies while the poor may be much more concerned about which inflation policy is adopted than the rich. A welfare-neutral policy may better balance the demands from the two sides.*

6 Related literature

While it has never been a mainstream proposition, that inflation may be expansionary can be at least dated back to Hume,

> ...[I]t is of no manner of consequence, with regard to the domestic happiness of a state, whether money be in a greater or less quantity. The good policy of the magistrate consists only in keeping it, if possible, still increasing; because, by that means he keeps alive a spirit of industry in the nation. [Hume (1752, p. 288)]

Hume, however, did not spell out why increasing the quantity of money may keep alive a spirit of industry. In fact, inflation tends to reduce output because it undercuts people's incentives to obtain money in most familiar models. Nonetheless there are models in line with Hume's proposition. In the presence of capital, the negative incentive effect of inflation on output may be dominated by the Tobin effect; see Orphanides and Solow (1990) for a survey. Moreover, inflation may be expansionary when agents have nonstandard preferences; e.g., Graham and Snower (2008). Furthermore, it is well known that with nominal rigidity, inflation can raise output as in the New Keynesian model; see, e.g., Devereux and Yetman (2002) and Levin and Yun (2007). In our model, the price is flexible and preferences are standard and, what kind of output-inflation correlation would emerge depends on how inflation redistributes wealth among agents.

It is not a mainstream proposition that monetary policy in general and inflation in specific would play a major role in shaping inequality in the long run, either. Nonetheless, three stylized facts in the U.S. economy seem to draw a fair amount of attention from the literature: poor people conduct larger proportions of transactions by cash; poor people hold larger proportions of wealth in cash; and only a fraction of households hold financial accounts. Erosa and Ventura (2002) formulate the first heterogeneous-agent model to endogenize the first two facts by assuming that some agents are more productive than others and that paying by some non-cash method is more costly than paying by cash; inflation in their model is effectively a regressive consumption tax. Motivated by the third fact, Williamson (2008) assumes that some agents cannot receive money transfers from the government. As such, inequality grows with inflation. In our model, all transactions are paid by money, access to the financial market and the money-transfer program is free, while inflation can easily be regressive to shift the distribution by a large degree when agents are *ex ante* identical.

Models with heterogeneous agents are employed to address welfare implications of inflation. For example, İmrohoroğlu (1992), Camera and Chien (2014), and Dressler (2011) quantify *ex ante* welfare costs of inflation due to lump sum transfers in different versions of the Bewley model; Molico (2006) does so in the Trejos-Wright-Shi model with $\theta = 1$; and Chiu and Molico (2010, 2011) do so in a model that mixes the Trejos-Wright-Shi model with the Lagos-Wright model (it is costly for agents to participate in the competitive market after they trade in pairs). Complementary to their works, our paper emphasizes that *ex ante* welfare costs may critically depend on the underlying inflation policy and the market structure. Moreover, our paper quantifies *ex post* welfare costs for an individual agent according to his wealth status if inflation comes as an unanticipated shock.

Some models with heterogeneous agents are designed to obtain analytical tractability by making certain assumptions on preferences and the market structure. For example, Boel and Camera (2009) introduce two types of agents who permanently differ in productivity into a version of the Lagos-Wright model; Menzio et al. (2013) separate the centralized labor market from the directed-search goods market; Rocheteau et al. (2018) formulate a continuous-time version of the Bewley model in which agents continuously consume and produce with a quasi-linear preference while being randomly hit by a preference shock for lumpy consumption; Rocheteau et al. (2021) study a version of the model of Berentsen et al. (2011) in which agents inelastically supply labor when meeting firms; and Lippi et al. (2015) consider a model in which two types of agents randomly switch their types (*à la* Levine 1991). Those models are quite useful in yielding certain insights. For example, Rocheteau et al. (2018) demonstrate that regressive policies dominate progressive policies when agents have sufficient capacity to self insure; Rocheteau et al. (2021) show that transferring money to firms and worker have different implications on the long-run Phillips curve; and Lippi et al. (2015) feature an optimal monetary policy that depends on aggregate states. As is well known, the Trejos-Wright-Shi model is not tractable; we use it because of its distinct feature that agents earn their labor income (with elastic labor supply) entirely by decentralized trade (through bilateral bargaining). This feature seems to drive the quantitative implications of our model.

Recently, monetary economics explores the influence of heterogeneity on the performance of the aspects of policy responding to economic cycles. The dominant framework of those works is the heterogeneous-agent New Keynesian model, a model that blends the basic ingredients of the standard New Keynesian model with the Bewley model; see Kaplan and Violante (2018) for a comprehensive review. Insistence on nominal rigidity reflects the dominant view of the profession; that is, a change in a nominal object such as the stock of money or the nominal interest rate would be irrelevant absent of sticky prices. Different from this strand of the New-Keynesian literature, our paper focuses on the long run aspects of policy. Our paper demonstrates that with decentralized trade, a change in a nominal object can be rather significant in the long run absent of any imposed nominal rigidity, a very similar message delivered by Jin and Zhu (2019) in a context for the short-run change.

7 Concluding remarks

This paper presents two findings regarding the long-run real effects of inflation. First, the real effects of inflation depend on the nature of inflation policy. Second, the real effects also depend on the market structure; in particular, decentralized trade (earning and spending labor income by bilateral bargaining) can have much different implications from centralized trade.

Individual risk is central to our explanation of these two findings. Three important factors may affect the individual risk but are absent in our study. The first is persistence in the idiosyncratic shock. We may let the productivity of an agent as a seller be determined by an idiosyncratic shock and the shock follows, say, an AR(1) process. Such a setting should further increase the individual risk. The second is a social safety net. Within the current setting, we may interpret ω in the utility function (see (2)) as a universal-consumption subsidy and choose the level of ω equal to a pre-chosen fraction $\bar{\omega}$ of the average consumption in the zero-inflation steady state. If $\bar{\omega} = 25\%$, then $\omega = 0.22$ and the risk aversion Σ is 0.84, sufficient to maintain main patterns of the inflation influence on output and the distribution. The third factor is intrinsic heterogeneity. We may add to the model a small class of agents who are more productive (as sellers) or more patient or both and, hence, richer overall. Likely, the addition of this rich class would increase the individual risk for agents in the non-rich class because the non-rich class only occupies a share of wealth to insure against their risks. This conjecture, of course, requires some careful check.

Finally, one may replace bilateral bargaining in the Trejos-Wright-Shi model with directed search. In this alternative environment of decentralized trade, buyers and sellers choose to visit submarkets indexed by price. It is for the future research to sort out whether the endogenized risks on consumption and production can still be sufficiently amplified.

Appendix A: Complete description of equilibria

A.1 The basic model

Under a transfer policy (C, C_0) in section 2, the expected amount of money received by an agent holding m units of money is $x(m) = \min\{\max\{0, C_0 + C \cdot m\}, B - m\}$. Let $\lfloor x(m) \rfloor$ be the largest integer no greater than $x(m)$; let $\lceil x(m) \rceil$ denote the smallest integer no less than $x(m)$ but no greater than $B - m$. If $\lceil x(m) \rceil \neq \lfloor x(m) \rfloor$, then $\lambda_t(m', m)$ is defined by

$$\begin{aligned} \lambda\left(m + \lfloor x(m) \rfloor, m\right) &= \lceil x(m) \rceil - x(m), \\ \lambda\left(m + \lceil x(m) \rceil, m\right) &= m - \lfloor x(m) \rfloor; \end{aligned}$$

and if $\lceil x(m) \rceil = \lfloor x(m) \rfloor$, then $\lambda_t(m', m)$ is defined by

$$\lambda\left(m + \lfloor x(m) \rfloor, m\right) = 1.$$

In a stage-2 meeting between a buyer with m^b and a seller with m^s, the equilibrium trading outcome $\mu(m^b, m^s)$ implies that

$$\begin{aligned}
\hat{\lambda}_t^b(m^b - d, m^b, m^s) &= \mu(d; m^b, m^s), \\
\hat{\lambda}_t^s(m^s + d, m^b, m^s) &= \mu(d; m^b, m^s),
\end{aligned}$$

where $d \in \{0, 1, ..., \min\{B - m^s, m^b\}\}$.

A.2 The model with centralized trade

Consider the version of the model with a centralized market in stage-2. Given the trading outcome $(y_t^a(m), \mu_t^a(.; m))$ (determined by (17)) and the distribution prior to the market $\hat{\pi}_t$, the value for an agent holding m right prior to stage-2 market is

$$\hat{v}_t(m) = \tilde{v}_t(m) + 0.5 \sum_{m'} \hat{\pi}_t(m') \left[S_t^b\left(y_t^b(m, m'), \mu_t^b(m, m'), m\right) + S_t^s\left(y_t^s(m', m), \mu_t^s(m', m), m\right) \right]; \tag{28}$$

the proportion of agents holding m right prior to date-t disintegration of money is

$$\tilde{\pi}_t(m) = 0.5 \sum_{m'} \left[\hat{\lambda}_t^b(m, m') + \hat{\lambda}_t^s(m, m') \right] \hat{\pi}_t(m'), \tag{29}$$

where $\hat{\lambda}_t^b(m, m')$ and $\hat{\lambda}_t^s(m, m')$ are the proportion of buyers with m' and the proportion of sellers with m', respectively, leaving the market with m; they are given by

$$\begin{aligned}
\hat{\lambda}_t^b(m^b - d^b, m^b) &= \mu^b(d^b; m^b), \\
\hat{\lambda}_t^s(m^s + d^s, m^s) &= \mu^s(d^s; m^s),
\end{aligned}$$

where $d^b \in \left\{0, 1, ... m^b\right\}$ and $d^s \in \{0, 1, ..., B - m^s\}$.
Given π_0, a sequence $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ is an *equilibrium* if it satisfies (3), (4), (11), (12), (18), (28), and (29), all t. A tuple (v, π, ϕ) is a steady state if $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ with $(v_t, \pi_{t+1}, \phi_t) = (v, \pi, \phi)$ all t is an equilibrium.

A.3 The model with nominal bonds

Under a hybrid policy with active transfer ($K > 0$), the expected amount of money transfer received by an agent with portfolio ζ is $\tilde{x}(\zeta) = \min\{K(1 + \zeta_2)^{-1}, B - \zeta_1 - \zeta_2\}$. Let $\lfloor \tilde{x}(\zeta) \rfloor$ denote the largest integer no greater than $\tilde{x}(\zeta)$; let $\lceil \tilde{x}(\zeta) \rceil$ denote the smallest integer no less than $\tilde{x}(\zeta)$ but no greater than $B - \zeta_1 - \zeta_2$. If $\lceil \tilde{x}(\zeta) \rceil \neq \lfloor \tilde{x}(\zeta) \rfloor$, then $\tilde{\mu}(.; \zeta)$ is defined by

$$\begin{aligned}
\tilde{\mu}\left(\lfloor \tilde{x}(\zeta) \rfloor, \zeta\right) &= \lceil \tilde{x}(\zeta) \rceil - \tilde{x}(\zeta), \\
\tilde{\mu}\left(\lceil \tilde{x}(\zeta) \rceil, \zeta\right) &= \tilde{x}(\zeta) - \lfloor \tilde{x}(\zeta) \rfloor;
\end{aligned}$$

and if $\lceil \tilde{x}(\zeta) \rceil = \lfloor \tilde{x}(\zeta) \rfloor$, then $\tilde{\mu}(.;\zeta)$ is defined by

$$\tilde{\mu}\left(\lfloor \tilde{x}(\zeta) \rfloor, \zeta\right) = 1.$$

Appendix B: Computation

Here we begin with the algorithm to compute steady states of the basic models.

1. Begin with an initial guess (π^0, v^0), where π^0 is consistent with the total money stock M, and v^0 is strictly concave.
2. Given (π^i, v^i), we follow the sub-steps below to update (π^{i+1}, v^{i+1}) and obtain $(\hat{\pi}^{i+1}, \tilde{\pi}^{i+1}, \hat{v}^{i+1}, \tilde{v}^{i+1})$.

   (a) Given π^i, we obtain $\hat{\pi}^{i+1}$ by (3) and δ^{i+1} by $\delta^{i+1} = 1 - M/\left(\sum m\hat{\pi}^{i+1}(m)\right)$.

   (b) Given δ^{i+1} and v^i, $\tilde{v}^{i+1}$ is determined by (4).

   (c) Given $\hat{\pi}^{i+1}$ and $\tilde{v}^{i+1}$, we solve the problem in (5) and obtain $\hat{v}^{i+1}$ from (9) and $\tilde{\pi}^{i+1}$ from (10).

   (d) Given $\tilde{\pi}^{i+1}$, $\hat{v}^{i+1}$, and δ^{i+1} computed in step (a), we obtain v^{i+1} from (11) and π^{i+1} from (12).

3. Repeat step 2 until $\min\left\{\|v^{i+1} - v^i\|,\ \|\pi^{i+1} - \pi^i\|\right\} < \epsilon$, where $\epsilon = 10^{-8}$.
4. Denote by (π^*, v^*) the final result.[19]

The steady-state algorithm for the model in section 3 with centralized market is similar. The only difference is in step 2, where we have to solve problems in (17) and (18) for all m^b and m^s, respectively; we also have to find an equilibrium price ϕ^i that clears the centralized market. The steady-state algorithm for the model in section 4 with nominal bonds can also be adapted in a straightforward manner.

[19]The accompanying FORTRAN 90 codes for the algorithms are available upon request. For $\theta = 0.98$, applying parallel computing on a server with a 48-thread CPU takes less than half a minute to converge; on a laptop with an Intel i7 CPU without parallel computing, it takes approximately 30 minutes. Convergence is fastest for $\theta = 1$: 4 minutes on the laptop. A small θ can demand much more time when it requires that B be significantly above 150 to mitigate the effect of bounding one's nominal wealth. For $\theta = 0.5$, $B = 900$ and convergence takes 2 hours on the server.

		benchmark	regressive transfer				progressive transfer			
		Σ	ΔY	ΔY_π	ΔV	$\Delta Gini$	ΔY	ΔY_π	ΔV	$\Delta Gini$
baseline		1.48	4.94%	4.32%	−1.40%	0.07	−2.76%	2.38%	0.27%	0.05
σ	0.5	0.81	0.60%	0.37%	−0.74%	0.03	−3.36%	1.43%	−0.24%	0.08
	1.5	1.90	8.25%	5.53%	−7.96%	0.00	−39.89%	10.80%	11.64%	0.02
η	0.25	1.84	3.08%	3.05%	−4.33%	0.11	−2.22%	−0.76%	1.49%	−0.04
	4	1.28	6.80%	5.67%	−1.12%	0.05	−4.19%	5.59%	0.20%	0.06
ω	10^{-6}	1.66	10.04%	9.34%	−2.11%	0.09	−4.07%	2.05%	0.44%	0.03
	10^{-2}	1.19	1.90%	1.62%	−1.07%	0.05	−2.31%	1.96%	0.17%	0.06
F	1	1.50	4.83%	2.64%	−1.05%	0.03	−8.67%	12.85%	0.01%	0.13
	365	1.47	5.01%	4.43%	−1.42%	0.07	−0.36%	0.78%	0.35%	0.06

Table 4: Effects of regressive and progressive transfer under various $(\sigma, \eta, \omega, F)$.

As noted in the main text, Molico (2006) numerically solves the divisible-money setup in footnote 5 with $\theta = 1$. An algorithm to solve the divisible-money setup needs (a) a $B' < \infty$ to approximate $B = \infty$; (b) a finite grid to approximate divisible money; and (c) in each iteration, a large number of samples from a given distribution to approximate that distribution. These approximations are saved in our model.

Appendix C: Robustness Check

For Table 4, recall that the baseline value of $(\sigma, \eta, \omega, F)$ is $(1, 1, 10^{-4}, 4)$ and that we change one parameter value at a time. In the table, the regressive transfer has $(C, C_0) = (0.01, -0.01)$ ($\varphi = 1\%$) and the progressive transfer has $(C, C_0) = (0, 0.3)$ ($\varphi = 1\%$) when we vary σ, η, and ω. When varying F, we adjust (C, C_0) for each transfer proportionally to F to keep the quarterly inflation rate at 1%. In the table, a progressive transfer undercuts *ex ante* welfare ($\Delta V = -0.24\%$) with $\sigma = 0.5$. This does not contradict the fact that progressive transfer improves welfare at low inflation; indeed, $\Delta V = 0.07\%$ when φ= 0.1% (i.e., $(C, C_0) = (0, 0.03)$).

For Table 5, recall that the baseline value of $(\sigma, \eta, \omega, F)$ is $(1, 1, 10^{-4}, 4)$ and we change either σ or η.

		$\theta = 1$					$\theta = 0.8$				
		Σ	regressive		progressive		Σ	regressive		progressive	
			ΔY	ΔV	ΔY	ΔV		ΔY	ΔV	ΔY	ΔV
baseline		1.08	1.41%	−0.46%	−2.13%	0.03%	1.22	5.27%	−3.11%	−16.08%	5.24%
σ	0.5	0.79	0.55%	−0.72%	−3.22%	−0.21%	0.69	0.55%	−0.37%	−3.68%	−0.82%
	1.5	1.40	16.34%	−0.41%	−1.84%	0.01%	1.79	4.02%	−7.34%	−49.52%	17.25%
η	0.25	1.10	0.10%	−0.28%	−0.81%	0.01%	1.33	0.49%	−4.46%	−7.27%	10.38%
	4	1.07	3.68%	−0.65%	−3.57%	0.05%	1.13	10.34%	−2.21%	−14.73%	1.63%
		$\theta = 0.6$					$\theta = 0.4$				
		Σ	regressive		progressive		Σ	regressive		progressive	
			ΔY	ΔV	ΔY	ΔV		ΔY	ΔV	ΔY	ΔV
baseline		0.94	1.76%	−0.38%	−19.37%	−2.72%	0.51	0.68%	0.23%	−15.08%	−7.39%
σ	0.5	0.45	0.49%	0.07%	−10.05%	−4.08%	0.37	0.65%	0.28%	−15.52%	−8.92%
	1.5	1.56	4.41%	−4.78%	−60.44%	9.76%	0.56	0.65%	0.21%	−14.60%	−6.66%
η	0.25	1.13	0.66%	−1.14%	−15.68%	−0.33%	0.53	0.49%	0.22%	−12.35%	−6.87%
	4	0.79	1.86%	−0.14%	−20.57%	−3.36%	0.48	0.81%	0.24%	−17.73%	−7.45%

Table 5: Impact of varying θ under various (σ, η).

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