Mundell II Revisited: A Mechanism-Design Foundation for Currency Unions*

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Abstract

Mundell II argues that a fixed exchange-rate regime (currency union) can improve welfare by promoting international risk sharing when aggregate shocks are asymmetric and labor is immobile. This paper reexamines Mundell II in a two-country monetary model by comparing a fixed exchange-rate regime with a flexible exchange-rate regime. Using a mechanism-design approach, the Kareken–Wallace indeterminacy is eliminated endogenously. Fixing exchange rates facilitates risk sharing but may also reduce the unified future value of currencies relative to their separate values. Mundell II holds for a class of power utility functions, but for general preferences, neither regime is unambiguously optimal.

Key words: Flexible prices; exchange-rate regimes; unified currency; optimal currency areas; neutrality

*This paper isolates and develops material in Section 4 (Decentralized stage-2 markets) of a working paper entitled "On the Nonneutrality of the Exchange-Rate Regime." The present version reorganizes that analysis around a focused reexamination of Mundell II.

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1 Introduction

More than twenty-five years after its launch, the euro has become a resounding success in terms of public acceptance, with support reaching record highs in recent surveys (Dreher et al. [3]). Yet academic debate persists on whether the euro area constitutes an optimal currency area. Critics, both in early skeptical writings (Eichengreen [4]; Feldstein [5]) and more recent analyses (Beck and Okhrimenko [1]; Grimm et al. [9]), continue to invoke optimal-currency-area (OCA) theory of Mundell [17], namely, asymmetric shocks, limited labor mobility, and the absence of substantial fiscal transfers, to argue that the euro area falls short of traditional optimality criteria. These recent studies acknowledge the endogeneity of OCA criteria (Frankel and Rose [7]); that is, by fostering trade and cycle synchronization, a currency union potentially makes it optimal *ex post*. The finding, however, is that endogeneity has been limited in the euro area, with persistent asymmetries.

Popular support alongside enduring theoretical concerns motivates a closer look at Mundell's own argument for the euro. As clarified by McKinnon [16], Mundell [18] advanced a distinct view—Mundell II, purporting that a common currency can actually enhance welfare by improving international risk-sharing when conventional OCA conditions are not met. Mundell II thus provides a potential microfounded rationale for why the euro might succeed despite theoretical concerns.

The Mundell II argument has remained a minority view. To resolve the Kareken–Wallace [12] indeterminacy, early attempts (e.g., Ching and Devereux [2]) rely on auxiliary assumptions such as currency-specific utility weights or cash-in-advance constraints. These assumptions mechanically favor the unified currency once multiple monies are eliminated, raising doubts about whether the risk-sharing benefit survives a cleaner modeling of multiple currencies.

This paper provides the first mechanism-design examination of Mundell II that endogenously resolves the Kareken–Wallace [12] indeterminacy without imposing exogenous home-currency biases. In a two-country version of the Lagos and Wright [14] model with aggregate asymmetric shocks, I compare flexible exchange rates with fixed exchange rates (equivalent to a unified currency) under their respective optimal incentive-compatible trading protocols. Domestic trade occurs in pairwise meetings, where portfolio-dependent core selection (Zhu and Wallace [26]; Hu et al [10]) endogenizes no currency substitution and supports high output.

I find two opposing effects of fixing exchange rates. The risk-sharing effect—the core of Mundell II—arises when the unified future real value of currencies suffices to incentivize efficient current production, while separate future values under flexible rates fall short in the unlucky country. The countervailing value-losing effect emerges when current relative marginal utilities and costs differ across countries: flexible rates would assign higher value to the currency of the currently attractive country—the country with the higher current consumption–production surplus, but a fixed rate forces a common value determined partly by the less attractive country, reducing the overall real return on money holdings. For power utility functions, pairwise consumption–production conditions are identical across countries at the efficient allocation, eliminating value loss. For general preferences, however, neither regime dominates.

Mundell II, therefore, offers a partial microfounded justification for currency unions such as the euro. True, a common currency provides an insurance channel absent under pure floats, but it also introduces an endogenous value-losing cost: a common real return dragged down by the worse-off country's fundamentals. This theoretical trade-off may be mapped to the real world. The insurance channel delivers tangible benefits and stability, contributing to the popular support for the euro. At the same time, the value-losing effect echoes familiar "one-size-fits-all" critiques of ECB monetary policy during periods of divergence. On a general level, this paper revisits the theory of currency unions from a monetary perspective in the spirit of Mundell [18], providing a complement to the recent fiscal-integration approach emphasized by Farhi and Werning [6] and rooted in the tradition of Kenen [13].

2 The model

There are two countries, 1 and 2. Each is populated by a nonatomic unit measure of infinitely-lived people and has its own currency. Each date has two stages; each stage has a produced and perishable good. Each person enters a date with a portfolio $m = (m_1, m_2) \in \mathbb{R}^2_+$, where m_k is the amount of currency k, i.e., country k's currency, for $k \in \{1, 2\}$. Then, three shocks, two idiosyncratic shocks and one aggregate shock, are realized. One idiosyncratic shock determines a person to be a *producer* or a *consumer* at stage 2 for that date with equal probability. Another determines a constant fraction

λ of consumers in each country to be *tourists* at stage 2 for that date.[1] The aggregate shock determines the current aggregate state; there are I aggregate states. The transition of states follows a Markov chain with a positive *transition matrix* $\pi = (\pi_{ij})$.

At stage 1, everyone can produce and consume a linear good, i.e., one's utility from consuming q is q and from producing q is $-q$. At stage 2, a producer produces and a nontourist consumer consumes in the home country; a tourist consumes in the foreign country and returns home at the end of the date; and real international trade consists solely of tourism. When the current (aggregate) state is $i \in \{1, 2, ..., I\}$, the utility of a consumer who consumes $q \geq 0$ in country k is $u_{ik}(q) \equiv \theta_{ik}u(q)$ and the disutility of a producer who produces q in country k is $c_{ik}(q) \equiv \rho_{ik}c(q)$, where $\theta_{ik}, \rho_{ik} > 0$, $u(0) = c(0) = 0$, $u' > 0$, $u'' < 0$, $c' > 0$, $c'' \geq 0$, $\beta u'(0) > c'(0)$, and β is the discount factor. Each person's period utility is the sum of his stage-1 utility and stage-2 utility; he maximizes expected discounted utility. Refer to $\alpha = (\theta_{i1}, \rho_{i1}, \theta_{i2}, \rho_{i2})_{i=1}^{I}$ as the *shock vector* and represent the aggregate shock by (π, α).

Stage 1 has a worldwide competitive market for everyone to trade the two currencies and the linear good. where the exchange rate, i.e., the relative price of two currencies, is either flexible or fixed. When the exchange rate is fixed, as is standard, the government of each country is committed to supply unlimited amounts of its own currency. I normalize the fixed value of the exchange rate as unity.

Stage 2 has a domestic market in each country to trade the stage good, where each consumer randomly meets a producer in each stage-2 domestic market. For a pairwise meeting, let (q, κ, ι) denote a generic outcome, where q is the producer's output, κ is the consumer's payment in the producer's home currency (i.e., currency k), and ι is the payment in the producer's foreign currency; the consumer who holds the portfolio m and the producer who holds m' follow a trading rule, denoted f, to select an outcome

$$f_{ik}(m, m') \equiv (y_{ik}(m, m'), \kappa_{ik}(m, m'), \iota_{ik}(m, m')). \tag{1}$$

3 Equilibrium

I limit consideration to stationary equilibria in that when i is the current state, the amount of goods ϕ_{ik} spent in the current stage-1 market to acquire one unit of

[1]The tourism shock introduces minimal real international trade. As it turns out, it does not affect the main results.

currency k depends only on i. Refer to $\phi = (\phi_{i1}, \phi_{i2})_{i=1}^{I}$ as a stage-1 price vector. The (gross expected) rates of return from carrying currency k into the coming stage-1 market is

$$\zeta_{ik} = E_i(\phi_{jk}/\phi_{ik}),$$

where E_i stands for the expectation made at state i. As is well known, linearity of the stage-1 good implies that the continuation payoff $w_{ik}(m)$ for a country-k resident who leaves the current stage 2 with the portfolio m takes the form

$$w_{ik}(m) = \beta(m_1\phi_{i1}\zeta_{i1} + m_2\phi_{i2}\zeta_{i2}) + A_{ik} \tag{2}$$

for some constant A_{ik}; moreover, by (2), ζ_{ik} cannot exceed $1/\beta$ in any equilibrium.

I further concentrate on equilibria satisfying *no currency substitution*, which is represented by that at the end of stage 1 of each period, currency k is only carried by people who trade in country k at stage 2. It is convenient to denote by $\eta(b_k)$ a portfolio $b = (b_1, b_2)$ of currencies with $b_l = 0$, $l \neq k$. Let $\eta(m'_{ik})$ and $\eta(m_{ik})$ be the portfolio held by producers in country k and the portfolio held by consumers who will consume in country k after stage-1 trade, respectively. Given the price vector ϕ and the trading rule f(see (1)), $\eta(m'_{ik})$ and $\eta(m_{ik})$ must be the best response to each other, i.e.,

$$\begin{aligned}\eta(m_{ik}) \in \arg\max_{m=(m_1,m_2)} \{&-m_k\phi_{ik} - m_l\phi_{il} + u_{ik}(y_{ik}(m, \eta(m'_{ik}))) \\ &+ \beta[m_k - \kappa_{ik}(m, \eta(m'_{ik}))]E_i\phi_{jk} + \beta[m_l - \iota_{il}(m, \eta(m'_{ik}))]E_i\phi_{jl}\}\end{aligned} \tag{3}$$

and

$$\begin{aligned}\eta(m'_{ik}) \in \arg\max_{m'=(m'_1,m'_2)} \{&-m'_k\phi_{ik} - m'_l\phi_{il} - c_{ik}(y_{ik}(\eta(m_{ik}), m')) \\ &+ \beta[m'_k + \kappa_{ik}(\eta(m_{ik}), m'))]E_i\phi_{jk} + \beta[m'_l + \iota_{il}(\eta(m_{ik}), m'))]E_i\phi_{jl}.\end{aligned} \tag{4}$$

It is convenient to normalize the stock of currency k as 0.5 under the flexible exchange-rate regime (the flexible-rate regime hereafter), and the stock of two currencies as unity under the fixed exchange-rate regime (the fixed-rate regime hereafter). So for the stage-1 market clearing, the money-bond regime requires

$$m_{ik} + m'_{ik} = 1, \tag{5}$$

and the money-only regime requires

$$\sum\nolimits_k (m_{ik} + m'_{ik}) = 2. \tag{6}$$

The discipline imposed on the trading rule f is that the trading outcome it assigned to any meeting is in the meeting-specific pairwise core—notice that the core is well defined given the price vector ϕ.

Definition 1 *A flexible-rate (fixed-rate, resp.) equilibrium is a pair of a positive stage-1 price vector ϕ and a stage-2 trading rulef such that (3) and (4) are satisfied when* (5) *holds all (i, k) (* (6) *and $\phi_{i1} = \phi_{i2}$ hold all i, resp.) and that for every state i, country k, and portfolios (m, m'), the outcome $f_{ik}(m, m')$ lies in the meeting-specific pairwise core (defined using the continuation values implied by the price vector ϕ).*

When comparing the two exchange-rate regimes, our welfare criterion weights all people equally at the initial date prior to stage-1. With linearity of the stage-1 good , welfare then depends only on consumption and production at stage 2. I refer to a vector $y = (y_{i1}, y_{i2})_{i=1}^{I} \in \mathbb{R}_{+}^{2I}$ as an *allocation*, where y_{ik} is stage-2 output of country k in state i. I refer to the allocation y^* which is determined by

$$u'_{ik}(y^*_{ik}) = c'_{ik}(y^*_{ik}) \tag{7}$$

all (i, k) as the *efficient* allocation. An allocation y is an *equilibrium allocation* if it is supported by an equilibrium, i.e., there exists an equilibrium satisfying $y_{ik} = y_{ik}(m_{ik}, m'_{ik})$ all (i, k).[2]

Following Hu et al [10], I focus on parameter values of the discount factor β for which the efficient allocation can be an equilibrium allocation. The comparison of the two exchange-rate regimes comes down to comparing the corresponding parameter spaces of β.

Remark 1 A Definition-1 equilibrium can be implemented by the following game. In stage 1 of each period, all people play a Shapley-Shubik [22] market game. There are two trading posts. The stage-1 good is the numéraire and two currencies are traded in different posts. Under fixed exchange rates, prior to opening of the trading posts, the governments stand ready to exchange the two currencies at the fixed rate on demand.

[2]The tourism shock (fraction λ), which introduces real international trade, does not appear in any equilibrium condition. Indeed, it does not affect the monetary channels or results.

In stage 2 of each period, when a consumer holding m meets a producer holding m' in country k at state i, they have two rounds of moves. At round 1, the two persons simultaneously say *Yes* or *No*. If both say *Yes*, then they move to round 2; otherwise autarky is reached (i.e., $(q, \kappa, \iota) = (0, 0, 0)$) and the meeting is resolved. At round 2, the consumer proposes a trading outcome and then the producer says *Yes* or *No*. If Yes, then the trading outcome is the consumer's proposal; otherwise, the trading outcome is $f_{ik}(m, m')$.

Remark 2 The stage-2 game form is used in Hu et al. [10]; this game form is introduced by Zhu [25] for the same purpose, i.e., to implement a trading outcome in the pairwise core. When the mechanism designer—a planner with limited enforcement power—can be present in a finite number of meetings per period to act as an arbitrator, agents may call for arbitration if they wish to deviate from the prescribed outcome. In equilibrium, however, no agent ever calls for arbitration because the prescribed outcome already lies in the meeting-specific core.

4 Conditions to support the efficient allocation y^*

I begin with necessary conditions for an equilibrium (ϕ, f) to support the efficient allocation y^*. Let $u^*_{ik} = u_{ik}(y^*_{ik})$, $c^*_{ik} = c_{ik}(y^*_{ik})$, $d^*_{ik} = u^*_{ik} - c^*_{ik}$, $c^*_i = 0.5 \sum_k c^*_{ik}$, and $d^*_i = 0.5 \sum_k d_{ik}$.

Consider a consumer with $\eta(m_{ik})$ and a producer with $\eta(m'_{ik})$ in a country-k meeting at state i. The producer leaves stage 1 with $\eta(m'_{ik})$ only if

$$m'_{ik}\phi_{ik} \leq -c^*_{ik} + \beta[m'_{ik} + \kappa_{ik}(\eta(m_{ik}), \eta(m'_{ik}))]E_i\phi_{jk}; \tag{8}$$

that is, the (discounted) future value of all money accumulated by him at stages 1 and 2 must cover his cost of producing y^*_{ik} plus stage-1 cost to acquiring m'_{ik}. Also, the consumer leaves stage 1 with $\eta(m_{ik})$ only if

$$m_{ik}\phi_{ik} \leq u^*_{ik} + \beta[m_{ik} - \kappa_{ik}(\eta(m_{ik}), \eta(m'_{ik}))]E_i\phi_{jk}; \tag{9}$$

that is, his utility of consuming y^*_{ik} plus the future value of any unspend money must cover his stage-1 cost to acquiring m_{ik}. Summing over (8) and (9), we have

$$\phi_{ik} \leq \frac{d^*_{ik}}{m_{ik} + m'_{ik}} + \beta E_i\phi_{jk}; \tag{10}$$

that is, the consumer-producer joint meeting surplus plus the future value of $m_{ik}+m'_{ik}$ must cover their joint stage-1 cost to acquiring $m_{ik}+m'_{ik}$.

The subsequent analysis is to draw for each regime from condition (8) a lower bound on the future value $\beta E_i\phi_{jk}$ of currency k so that a country-k producer is willing to bear the cost c^*_{ik} of producing y^*_{ik} at stage 2, and from (10) an upper bound on $\beta E_i\phi_{jk}$ so that a consumer who is to consume in country k at stage 2 is willing to bear the cost $m_{ik}\phi_{ik}$ of acquiring m_{ik} units of currency k at stage 1.

For the lower bounds on $\beta E_i\phi_{jk}$, I first apply $\beta E_i\phi_{jk}\le\phi_{ik}$ (the rate of return of currency k cannot exceed $1/\beta$) to (8) so

$$c^*_{ik}\le\beta\kappa_{ik}(\eta(m_{ik}),\eta(m'_{ik}))E_i\phi_{jk}. \tag{11}$$

Because the payment $\kappa_{ik}(\eta(m_{ik}),\eta(m'_{ik}))$ is bounded above by m_{ik}, (11) gives rise to

$$c^*_{ik}\le\beta m_{ik}E_i\phi_{jk}; \tag{12}$$

that is, the future value of currency k carried by the consumer must cover the producer's stage-2 cost of production. Given (12), the stage-1 market-clearing conditions (5) and (6) require

$$c^*_{i1}\le\beta E_i\phi_{j1}, c^*_{i2}\le\beta(1+r_i)E_i\phi_{j1} \tag{13}$$

for the flexible-rate regime and

$$c^*_i\le\beta E_i\phi_{j1} \tag{14}$$

for the fixed-rate regime.

For the upper bounds on $\beta E_i\phi_{jk}$ implied by (10), I first consider the flexible-rate regime.

Lemma 1 *Let $\pi_{ij}(t)$ denote the t-step transition probability from state i to j. Let*

$$v_{ik}(\beta)=\sum_{t\ge1}\sum_j\beta^t\pi_{ij}(t)d^*_{jk}, \tag{15}$$

$$x^*_{ik}=d^*_{ik}+v_{ik}(\beta). \tag{16}$$

If y^* is supported by a flexible-rate equilibrium (ϕ,f), then $\phi_{ik}\le x^*_{ik}$ and $\beta E_i\phi_{jk}\le v_{ik}(\beta)$.

Proof. Suppose y^* is supported by a flexible-rate equilibrium (ϕ,f). Let a mapping $H : \{x = (x_{i1}, x_{i2})_{i=1}^{I} : x \geq \phi\} \to \mathbb{R}^{2I}$ be defined by

$$H_{ik}(x) = d_{ik}^* + \beta E_{ik} x_{jk}.$$

Apparently, H is a contraction mapping. By (10) and the stage-1 market clearing condition (5), $\phi_{ik} \leq d_{ik}^* + \beta E_i \phi_{jk}$ so $H(\phi) \geq \phi$. Because $H(x) \geq H(\phi)$ for $x \geq \phi$, it follows from the contraction-mapping theorem that H has a unique fixed point x°, i.e.,

$$x_{ik}^\circ = d_{ik}^* + \beta E_i x_{jk}^\circ \tag{17}$$

and $x^\circ \geq \phi$. By repeated substitution, (17) yields

$$\beta E_i x_{jk}^\circ = v_{ik}(\beta). \tag{18}$$

Comparing (16) with (17), we conclude from (18) that $x^\circ = x^*$. ■

Lemma 1 tells that for the flexible-rate regime, when i is the current state, x_{ik}^* is the maximal possible current value of currency k (i.e., the maximal possible current stage-1 price of currency k) and $v_{ik}(\beta)$ is the maximal possible future value of currency k. Notice that both x_{ik}^* and $v_{ik}(\beta)$ are exogenous because they are completely pinned down by the terms of d_{ik}^*. One might anticipate that the terms of d_{ik}^* also pin down the upper bound on $\beta E_i \phi_{jk}$ for the fixed-rate regime. This is the case but the upper bound takes a more complicated form.

Lemma 2 *There exist greatest vectors $\nu = (\nu_i)_{i=1}^I$ and $\delta = (\delta_i)_{i=1}^I$ that satisfy*

$$\nu_i(\beta) = \sum_{t \geq 1} \sum_j \beta^t \pi_{ij}(t) \delta_j \tag{19}$$

and

$$\delta_i = \min\{d_i^*, \max\{\nu_i(\beta), c_i^*\} \min_k (d_{ik}^*/c_{ik}^*)\}, \tag{20}$$

and $\nu_i(\beta)$ is strictly increasing and continuous in β. Let

$$z_i^* = \delta_i + \nu_i(\beta). \tag{21}$$

If y^* is supported by a fixed-rate equilibrium (ϕ, f), then $\phi_i \leq z_i^*$ and $\beta E_i \phi_j \leq \nu_i(\beta)$, where ϕ_j is the common value of ϕ_{j1} and ϕ_{j2}.

Proof. Existence of $\nu(\beta)$ and δ and properties of δ are shown in the appendix. Now suppose y^* is supported by a fixed-rate equilibrium (ϕ, ξ) and let a mapping

$G: \ \{z = (z_i)_{i=1}^I : z \geq \phi^\circ \equiv (\phi_i)_{i=1}^I\} \to \mathbb{R}^I$ be defined by

$$G_i(z) = \max_{(b_{i1}, b_{i2})} [\min_k (d_{ik}^*/b_{ik}) + \beta E_i z_j] \tag{22}$$
$$\text{s.t. } b_{i1} + b_{i2} = 2 \text{ and } c_{ik}^* \leq \beta b_{ik} E_i z_j \text{ all } k.$$

By (10) and the stage-1 market clearing condition (6), $\phi_i \leq \min_k(d_{ik}^*/m_{ik}) + \beta E_i \phi_j$ so $G(\phi^\circ) \geq \phi^\circ$. Because $G(z) \geq G(z')$ for $z \geq z'$, it follows from the Tarski fixed-point theorem that G has a greatest fixed point z° and $z^\circ \geq \phi^\circ$. Let

$$\Delta_i = \min\{d_i^*, \beta E_i z_j^\circ \min_k (d_{ik}^*/c_{ik}^*)\} \tag{23}$$

and let b_{ik}° denote the optimal b_{ik} for the problem in (22) when $z = z^\circ$. We claim

$$\min_k (d_{ik}^*/b_{ik}^\circ) = \Delta_i, \tag{24}$$

which is verified in the appendix. By (24) and $G_i(z^\circ) = z_i^\circ$,

$$z_i^\circ = \Delta_i + \beta E_i z_j^\circ. \tag{25}$$

By repeated substitution, (25) yields

$$\beta E_i z_j^\circ = \sum_{t \geq 1} \sum_j \beta^t \pi_{ij}(t) \Delta_j. \tag{26}$$

By constraints in (22), $\beta E_i z_j^\circ \geq c_i^*$ so in particular $\beta E_i z_j^\circ = \max\{\beta E_i z_j^\circ, c_i^*\}$; plugging this into (23), we have

$$\Delta_i = \min\{d_i^*, \max\{\beta E_i z_j^\circ, c_i^*\} \min_k (d_{ik}^*/c_{ik}^*)\}. \tag{27}$$

Comparing (19) and (20) with (26) and (27), we conclude that $\beta E_i z_j^\circ = \nu_i$ and $\Delta_i = \delta_i$. Then comparing (21) with (25), we conclude that $z^\circ = (z_i^*)_{i=1}^I$. ■

Lemma 2 tells that for the fixed-rate regime, when i is the current state, $2z_i^*$ is the maximal possible current value of two currencies (which is also the twice of the maximal possible common stage-1 price of both currencies) and $2\nu_i(\beta)$ is the maximal possible future value of two currencies. A simple but useful observation from (20) is

$$\delta \leq d^* \equiv (d_i)_{i=1}^{*I}. \tag{28}$$

Comparing (15) and (16) with (19) and (21), one can further observe from (28) that

$$z_i^* \leq 0.5 \textstyle\sum_k x_{ik}^* \tag{29}$$

and

$$\nu_i(\beta) \leq 0.5 \sum\nolimits_k v_{ik}(\beta), \tag{30}$$

and the two inequalities become equalities iff $\delta = d$. As it turns out, (29) and (30) play the key roles in results present in the next section).

Now I can determine the values of the discount factor β that satisfy the lower bounds on $\beta E_i \phi_{jk}$ in (13) and (14) and the upper bounds on $\beta E_i \phi_{jk}$ in Lemmas 1 and 2. By (15), $v_{ik}(0) = 0$, $v_{ik}(1) = \infty$, and $v'_{ik}(\beta) > 0$; therefore, we have $\beta_x(i,k)$, a cutoff value of the discount factor for the flexible-rate regime, well defined by

$$v_{ik}(\beta_x(i,k)) = c^*_{ik}. \tag{31}$$

By (19) and Lemma 2, $\nu_i(0) = 0$, $\nu_i(1) = \infty$, and $v_i(\beta)$ is strictly increasing and continuous in β; therefore, we have $\beta_z(i)$, a cutoff value of the discount factor for the fixed-rate regime, well defined by

$$\nu_i(\beta_z(i)) = c^*_i. \tag{32}$$

Lemma 3 *Let* $\beta_x = \max_{(i,k)} \beta_x(i,k)$ *and* $\beta_z = \max_i \beta_z(i)$. *The efficient allocation* y^* *can be supported by the flexible-rate regime only if* $\beta \geq \beta_x$ *and by the fixed-rate regime only if* $\beta \geq \beta_z$.

Proof. By the aforementioned properties $v_{ik}(.)$, the flexible-rate regime can cover the cost of producing y^*_{ik}, i.e., (13) can be satisfied, only if $\beta \geq \beta_x(i,k)$; thus, y^* can be supported by the flexible-rate regime only if $\beta \geq \beta_x$. By the aforementioned properties $\nu_i(\beta) = 0$, the fixed-rate regime can cover the cost of producing $\sum y^*_{ik}$, i.e., (14) can be satisfied, only if $\beta \geq \beta_z(i)$; thus, y^* can be supported by the fixed-rate regime only if $\beta \geq \beta_z$. ■

Next I show that the Lemma-3 necessary conditions are sufficient. To this end, let me use the following problem to describe $(y_{ik}(m,m'), \kappa_{ik}(m,m'), \iota_{ik}(m,m'))$, the meeting outcome for a pairwise meeting in country k at state i between a consumer carrying m and a producer carrying m', selected by the trading rule f in the supporting equilibrium.

Problem 1 Fix $m^* \in \mathbb{R}^{2I}_{++}$ and proceed by two steps.

Step 1. Determine a meeting outcome $(\bar{y}_{ik}(m,m'), \bar{\kappa}_{ik}(m,m'), 0)$ as follows: if $m_k \geq m^*_{ik}$ then let

$$(\bar{y}_{ik}(m,m'), \bar{\kappa}_{ik}(m,m')) = \arg \max_{q\geq 0, 0\leq \kappa \leq m_k} [u_{ik}(q) - \beta\kappa E_i \phi_{jk}] \tag{33}$$

subject to $-c_{ik}(q) + \beta\kappa E_i \phi_{jk} \geq 0$; otherwise, let

$$(\bar{y}_{ik}(m,m'), \bar{\kappa}_{ik}(m,m')) = \arg \max_{q\geq 0, 0\leq \kappa \leq m_k} [-c_{ik}(q) + \beta\kappa_{ik} E_i \phi_{jk}] \tag{34}$$

subject to $u_{ik}(q) - \beta\kappa E_i \phi_{jk} \geq 0$.

Step 2. Let the meeting outcome assigned by the rule f be

$$f_{ik}(m,m') = \arg \max_{q\geq 0, 0\leq \kappa \leq m_k, 0\leq \iota \leq m_l} [-c_{ik}(q) + \beta E_i(\kappa\phi_{jk} + \iota\phi_{jl})] \tag{35}$$

subject to $u_{ik}(q) - \beta E_i(\kappa\phi_{jk} + \iota\phi_{jl}) \geq u_{ik}(\bar{y}_{ik}(m,m')) - \beta E_i \bar{\kappa}_{ik}(m,m')\phi_{jk}$.

The outcome $f_{ik}(m,m')$ determined by the step-2 optimization (35) in Problem 1 is in the pairwise core because there is no restriction on which currency can be used in payments.[3] This outcome maximizes the producer's payoff conditional on not making the consumer worse off than the trade $(\bar{y}_{ik}(m,m'), \bar{\kappa}_{ik}(m,m'), 0)$ obtained from the step-1 optimization.

Because of the restriction on the payment, the step-1 optimization turns the producer's home currency as the *right currency* for the meeting and, hence, endogenizes imperfect currency substitution (actually no substitution). Indeed, if the consumer does not carry the right currency, then the step-1 outcome is $(0,0,0)$ so that the step-2 optimization gives the producer all surplus from trade; this scheme is borrowed from Zhu and Wallace [26].[4]

Conditional on that the consumer carries only the right asset, the step-1 optimization assigns all surplus to the consumer if he carries a sufficient amount of right currency (sufficiency is measured by m^*_{ik}) and to the producer otherwise; borrowed from Hu et al. [10] and Hu and Rocheteau [11], this schemes encourages the consumer to spend a sufficient amount of real resources to acquire the right asset even when its

[3] Problem 1 does not represent an extensive game form with two rounds of alternating offers but it may be understood as a gradual bargaining problem (see O'Neill et al. [19]).

[4] One may obtain imperfect substitution of currencies by assuming that one currency differs from another in some fundamental aspect; for example, one currency is harder to counterfeit than another (see Gomis-Porqueras et al. [8] and Zhang [24] for related models). Our purpose here is to examine which regime is better when there is no currency substitution and when two currencies do not differ in any fundamental aspect; the Zhu and Wallace [26] scheme fits this purpose well.

rate of return is low.

Lemma 4 *The efficient allocation y^* is supported by the flexible-rate regime if $\beta \geq \beta_x$ and by the fixed-rate regime if $\beta \geq \beta_z$.*

Proof. For the flexible-rate regime, the supporting equilibrium (ϕ, f) has ϕ_{ik} equal to x^*_{ik} (see (16)) and $f$as the one in Problem 1 with $m^*_{ik} = 1$; for the fixed-rate regime, the supporting equilibrium (ϕ, f) has ϕ_i equal to z^*_i (see (21)) and $f$as the one in Problem 1 with $m^*_{ik} = c^*_{ik}/c^*_i$.

To confirm, consider the flexible-rate regime and the argument for the fixed-rate regime is similar. Fix a country-k meeting in state i between a consumer who resides in country l and carries $\eta(m_k)$ and a producer who carries $m' = (0, 0)$. Let q denote the output and κ the payment of currency k in the meeting outcome assigned by f.

If $m_k \geq 1$, then by (33), $c_{ik}(q) = \kappa v_{ik}(\beta)$ and $u'_{ik}(q) \geq c'_{ik}(q)$, strict only if $c^*_{ik} > m_k v_{ik}(\beta)$; by $\beta \geq \beta_x$, $v_{ik}(\beta) \geq c^*_{ik}$ so $q = y^*_{ik}$ and the consumer's payoff from the meeting outcome is $u^*_{ik} - c^*_{ik} + m_k v_{ik}(\beta) + A_{il}$ (see (2)). If $m_k < 1$ then by (34), the consumer's payoff from the meeting outcome is $m_k v_{ik}(\beta) + A_{il}$. Because the cost of carrying m_k into the meeting is $m_k \phi_{ik}$ and $\phi_{ik} > v_{ik}(\beta)$, it is optimal for the consumer to leave stage 1 with $m_k = 1$. ■

5 Results

When the two cutoff values β_x and β_z in Lemma 3 differ, one regime dominates another over a range of values of β—the fixed-rate regime dominates if $\beta_x > \beta_z$.

Proposition 1 *Suppose (i) given π, α is not in a measure-zero set in $\mathbb{R}^4$, and (ii) $u^*_{i1}/c^*_{i1} = u^*_{i2}/c^*_{i2}$ all i. Then $\beta_x > \beta_z$.*

Proof. By condition (ii), $d^*_i/c^*_i = d^*_{ik}/c^*_{ik}$ so $\max\{\nu_i(\beta), c^*_i\} \min_k(d^*_{ik}/c^*_{ik}) \geq d^*_i$. Then by (15), (19), and (20), we see $\delta = d$ and

$$\nu_i(\beta) = 0.5 \sum_k v_{ik}(\beta) \tag{36}$$

all i. Now fix i and let

$$\beta_x(i) = \max\{\beta_x(i, 1), \beta_x(i, 2)\}.$$

We claim that $\beta_x(i) \geq \beta_z(i)$ and $\beta_x(i) = \beta_z(i)$ only if $\beta_x(i,1) = \beta_x(i,2)$. As verified in the appendix, given π, $\beta_x(i,1) \neq \beta_x(i,2)$ when α is outside a measure-zero set in $\mathbb{R}^4$. So by condition (i) and the claim, $\beta_x(i) > \beta_z(i)$. Because i is arbitrary, $\beta_x > \beta_z$.

To verify the claim, suppose $\beta_z(i) \geq \beta_x(i)$. Then we have

$$c_i^* = \nu_i(\beta_z(i)) \geq \nu_i(\beta_x(i)) = 0.5 \sum_k v_{ik}(\beta_x(i)) \geq 0.5 \sum_k v_{ik}(\beta_x(i,k)) = c_i^*, \tag{37}$$

where the first equality uses the definition of $\beta_z(i)$ in (32), the first inequality uses strict monotonicity of $\nu_i(\beta)$ and it is strict if $\beta_z(i) > \beta_x(i)$, the second equality uses (36), the second inequality uses $v'_{ik}(\beta) > 0$ and it is strict if $\beta_x(i,1) \neq \beta_x(i,2)$, and the last equality uses the definition of $\beta_x(i,k)$ in (31). By (37), the claim must be true. ■

Proposition 1 is a formal statement of Mundell II. To understand Proposition 1, it helps to relate the term d_{ik} to the exogenous dividend in a version of the Lucas [15] asset-pricing model with risk neutrality. Specifically, at state i, d_{ik}^* is the dividend contributed by country k, d_i^* is the unintegrated worldwide dividend, and δ_i is the integrated worldwide dividend. In this connection, $v_{ik}(\beta)$ is solely determined by the dividend stream of country k, and the unified value $\nu_i(\beta)$ obtained by integrating the separated values $v_{i1}(\beta)$ and $v_{i2}(\beta)$ of two currencies is determined by the integrated worldwide dividend stream. Integration is costless if the integrated worldwide dividend is equal to the unintegrated worldwide dividend at all states (i.e., $\delta = d^*$) and is costly otherwise (i.e., $\delta \leq d^*$ and $\delta_j < d_j^*$ some j). Costless integration is ensured by condition (ii), a condition applicable to a class of functional forms of preferences (i.e., the functions u and c are power functions).

For the purpose of sustaining the efficient worldwide output, each country must rely on its own dividend stream to cover its production cost (i.e., $v_{ik}(\beta) \geq c_{ik}^*$) under the flexible-rate regime, while the two countries can share the integrated worldwide dividend stream to cover the worldwide production cost (i.e., $\nu_i(\beta) \geq c_i^*$) under the fixed-rate regime. Thus, by integrating $v_{i1}(\beta)$ and $v_{i2}(\beta)$ into $\nu_i(\beta)$, the fixed-rate regime promotes risk sharing between the two countries—it permits the worldwide efficiency to be sustained at state i in case that the dividend stream of country k is not sufficient (i.e., $v_{ik}(\beta) < c_{ik}^*$) but the integrated worldwide dividend stream is sufficient (i.e., $\nu_i(\beta) \geq c_i^*$). When integration is costless, we have (36), i.e., (30) holds in equality. Because existence of i satisfying $v_{ik}(\beta) < c_{ik}^*$ and $0.5\sum_k v_{ik}(\beta) < c_i^*$ is generic, the fixed-rate regime dominates thanks to the risk-sharing effect.

When integration is costly, there is *value losing* in that the inequality in (30) is strict, i.e.,

$$\nu_i(\beta) < 0.5\textstyle\sum_k \upsilon_{ik}(\beta). \tag{38}$$

To think of costly integration and the value-losing effect (38), let constraints in (10) and (12) be binding and $m'_{ik} = 0$; then (10) can be written as

$$\phi_{ik} = (u^*_{ik}/c^*_{ik})\beta E_i \phi_{jk}. \tag{39}$$

Refer to u^*_{ik}/c^*_{ik} as the country k's pairwise consumption-production condition at state i; this condition tells how much util gain from consumption can be obtained from per unit util cost of production in a country k's pairwise meeting at state i. Then (39) simply means that people are willing to pay a higher price for currency k in the stage-1 market at a state if country k has a better pairwise condition at that state. But fixing exchange rates forces the stage-1 price of each currency to depend on the worse of the two conditions at the state. So when the two pairwise conditions differ at a state (i.e., $u^*_{j1}/c^*_{j1} \neq u^*_{j2}/c^*_{j2}$ some j), some country's dividend at that state is lost (i.e., $\delta_j < d^*_j$). In general equilibrium, the loss of the current dividend passes to the future value of currencies, resulting in (38).

With costly integration, which regime dominates is ambiguous. It is best to see this by a simple example.

Example 1 *Let $I = 2$, $(\alpha_{11}, \alpha_{12}) = (\alpha_{22}, \alpha_{21})$ all i, and $\pi_{ij} = 0.5$ all (i, j).*

By Lemma 1, $\upsilon_{ik}(\beta) = \beta \bar{d}_k/(1-\beta)$, where $\bar{d}_k = 0.5(d^*_{1k} + d^*_{2k})$. By the structure of α, $\max_k c^*_{1k} = \max_k c^*_{2k}$ and $\bar{d}_k = d^*_1$. So by (31), $\max_k c^*_{1k} = \beta_x d^*_1/(1-\beta_x)$ or

$$(1-\beta_x)/\beta_x = \min_k (d^*_1/c^*_{1k}). \tag{40}$$

Also by the structure of α, c^*_i, d^*_i, $\nu_i(\beta)$, and $\min_k(d^*_{ik}/c^*_{ik})$ are all constant in i. So by Lemma 2,

$$\nu_i(\beta) = \beta \min\{d^*_i, \max\{\nu_i(\beta), c^*_i\} \min_k (d^*_{ik}/c^*_{ik})\}/(1-\beta);$$

then by (32), we have

$$c^*_1 = \beta_z \min\{d^*_1, c^*_1 \min_k (d^*_{1k}/c^*_{1k})\}/(1-\beta_z).$$

By $d_1^*/c_1^* \geq \min_k(d_{1k}^*/c_{1k}^*)$, this implies

$$(1-\beta_z)/\beta_z = \min_k(d_{1k}^*/c_{1k}^*). \tag{41}$$

It follows from that (40) and (41) that

$$\text{sign of } \beta_x - \beta_z = \text{sign of } \min_k(d_{1k}^*/c_{1k}^*) - \min_k(d_1^*/c_{1k}^*).$$

Proposition 2 *In Example 1, set* $u(q) = q - 0.5q^2$, $c(q) = q$, $\theta_{11} = \theta_{12} = 1$, *and* $\rho_{11} < \rho_{12} < 1$. *Then when* $0.5 > \rho_{12} > \rho_{11}$, $\beta_x < \beta_z$*; when* $\rho_{12} > \rho_{11} > 0.5$, $\beta_x > \beta_z$.

The proof of Proposition 2 is in the appendix. Example 1 consists of ingredients which would make risk sharing the dominant factor—two countries are *ex ante* identical, one country in one state mirrors another country in another state, and the aggregate shock is transitory. But the value-losing effect can actually dominate even when the country-specific risk is small (i.e., $|\rho_{11} - \rho_{12}|$ is small).

6 Concluding remark

Proposition 2 suggests that in general, neither regime would be the optimal way to manage exchange rates for worldwide production. But what is the optimal way? This calls for a formulation and analysis of a more general exchange-rate system—an issue that lies beyond the scope of the present paper and naturally motivates subsequent research.

Appendix

Completion of proof of Lemma 2

For existence of $\nu(\beta)$, let $\underline{\nu} = (\underline{\nu}_i)_{i=1}^I$ have $\underline{\nu}_i = \beta \min_j [c_j^* \min_k (d_{jk}^*/c_{jk}^*)]/(1-\beta)$ all i. Define a mapping $\nu \mapsto F(\nu)$ from the set $\{\nu = (\nu_i)_{i=1}^I : \nu \geq \underline{\nu}\}$ to $\mathbb{R}^I$ by

$$F_i(\nu) = \sum_{t\geq 1} \sum_{j=1} \beta^t \pi_{ij}(t) \min\{d_j^*, \max\{\nu_j, c_j^*\} \times \min_k (d_{jk}^*/c_{jk}^*)\}. \tag{42}$$

Note that for $\nu \geq \nu'$, $F(\nu) \geq F(\nu') \geq F(\underline{\nu})$. Then by the Tarski fixed-point theorem, F has a greatest fixed point, which is $\nu(\beta)$. For monotonicity of $\nu(.)$, let $\beta_2 > \beta_1$ and replace $\underline{\nu}$ with $\nu(\beta_1)$ in the domain of F and set $\beta = \beta_2$ in (42); then applying the Tarski fixed-point theorem once more, we get $\nu(\beta_2) > \nu(\beta_1)$. For continuity of $\nu(.)$, it suffices to consider the case that $\nu_j(\beta) = c_j^*$ for some j. For this case, it is clear that $\nu(\beta_n) \to \nu(\beta)$ as $\beta_n \uparrow \beta$. Setting $F_i(\nu) = \nu_i$, (42) can be written as linear equations $P(\beta)\nu = Q(\beta)$ in ν. Because the greatest fixed point of F is also the greatest solution to the linear equations $P(\beta)\nu = Q(\beta)$, $P(\beta)$ is invertible (otherwise there is a continuum of solutions, none of which can be the greatest). Therefore, $[P(\beta)]^{-1}Q(\beta) = \nu(\beta)$. When $\beta_n \downarrow \beta$, $\nu(\beta_n) = [P(\beta_n)]^{-1}Q(\beta_n) \to [P(\beta)]^{-1}Q(\beta)$.

For the claim $\min_k(d_{ik}/b_{ik}^\circ) = \Delta_i$, note either $b_{ik}^\circ \beta E_i z_j^\circ > c_{ik}^*$ for both k or not. If the former, then using $b_{ik}^\circ = b_{ik}^* \equiv d_{ik}^*/d_i^*$ and $b_{ik}^\circ \beta E_i z_j^\circ > c_{ik}^*$, we have $\beta E_i z_j^\circ (d_{ik}^*/c_{ik}^*) > d_i^* = d_{ik}^*/b_{ik}^\circ$, confirming the claim. So suppose without loss of generality that $b_{i1}^\circ \beta E_i z_j^\circ = c_{i1}^*$. Then $\beta E_i z_j^\circ (d_{i1}^*/c_{i1}^*) = d_{i1}^*/b_{i1}^\circ < d_{i1}^*/b_{i1}^* = d_i^*$ and $\beta E_i z_j^\circ (d_{i2}^*/c_{i2}^*) \geq d_{i2}^*/b_{i2}^\circ \geq d_{i2}^*/b_{i2}^* = d_i^*$, again confirming the claim.

Completion of proof of Proposition 1

To verify non-genericity of $\beta_x(i,1) = \beta_x(i,2)$, fix i and define the mapping $\alpha \mapsto \Gamma_k(\alpha)$ by $\Gamma_k(\alpha) = \sum_{t\geq 1}\sum_j \beta^t \pi_{ij}(t) d_{jk}^* - c_{ik}^*$, where y_{jk} is an implicit function of α determined by $c_{jk}'(y_{jk}) = u_{jk}'(y_{jk})$. Let $\Gamma(\alpha) = \Gamma_1(\alpha) - \Gamma_2(\alpha)$. Fix $j \neq i$. Using

$$\frac{\partial \Gamma}{\partial \theta_{j1}} = \sum_{t\geq 1} \sum_j \beta^t \pi_{ij}(t) [u(y_{j1}) + u_{j1}'(y_{j1}) - c_{j1}'(y_{j1})] \frac{\partial y_{j1}}{\partial \theta_{j1}},$$

we have

$$\frac{\partial \Gamma}{\partial \theta_{j1}} = \sum_{t\geq 1} \sum_j \beta^t \pi_{ij}(t) u(y_{j1}) \frac{\partial y_{j1}}{\partial \theta_{j1}}.$$

Therefore, it follows from

$$\frac{\partial y_{j1}}{\partial \theta_{j1}} = u'(y_{j1})[c''_{j1}(y_{j1}) - u''_{j1}(y_{j1})]^{-1}$$

that the Jacobian of Γ evaluated at any α has full rank. So the dimension of the zero set of Γ is $4I - 1$.

Proof of Proposition 2

Using $y^*_{1k} = 1 - \rho_{1k}$, we have $d^*_{1k} = 0.5(1 - \rho_{1k})^2$, $c^*_{1k} = \rho_{1k}(1 - \rho_{1k})$, $d^*_{1k}/c^*_{1k} = 0.5(1/\rho_{1k} - 1)$, and $d^*_1 = 0.25[(1 - \rho_{11})^2 + (1 - \rho_{12})^2]$. So $d^*_{11} > d^*_1 > d^*_{12}$ and $d^*_{11}/c^*_{11} > d^*_{12}/c^*_{12} = \min_k(d_{1k}/c^*_{1k})$. When $0.5 > \rho_{12} > \rho_{11}$, $c^*_{12} > c^*_{11}$ so $\min_k(d^*_1/c^*_{1k}) = d^*_1/c^*_{12}$. Thus $\beta_x < \beta_z$. When $\rho_{12} > \rho_{11} > 0.5$, $c^*_{11} > c^*_{12}$ so $\min_k(d^*_1/c^*_{1k}) = d^*_1/c^*_{11}$; moreover, $d^*_1/c^*_{11} > d^*_{12}/c^*_{12}$ iff

$$(1 - \rho_{11})^2 + (1 - \rho_{12})^2 > \frac{2\rho_{11}(1 - \rho_{11})(1 - \rho_{12})}{\rho_{12}}.$$

Set

$$\varsigma(a) = (1 - a)^2 + (1 - \rho_{12})^2 - \frac{2a(1 - a)(1 - \rho_{12})}{\rho_{12}}$$

for $a \leq \rho_{12}$. Now $d^*_1/c^*_{11} > d^*_{12}/c^*_{12}$ follows from $\varsigma(\rho_{12}) = 0$ and $\varsigma'(a) < 0$. Thus $\beta_x > \beta_z$.

References

[1] Beck, Krzysztof and Iana Okhrimenko, Optimum Currency Area in the Eurozone, *Open Economies Review* 36 (2025), 197-219.

[2] Ching, Stephen and Michael B. Devereux, Mundell Revisited: a Simple Approach to the Costs and Benefits of a Single Currency Area, *Review of International Economics* 11 (2003), 674–691.

[3] Dreher, Ferdinand, Niklas Jütting and Hanni Schölermann, Hitting Record Highs: Unpacking Support for the Euro, *ECB Economic Bulletin* 7 (2025).

[4] Eichengreen, Barry, European Monetary Unification, *Journal of Economic Literature* 3 (1993), 1321–1357.

[5] Feldstein, Martin, The Political Economy of the European Economic and Monetary Union: Political Sources of an Economic Liability, *Journal of Economic Perspectives* 4 (1997), 23–42.

[6] Farhi, Emmanuel and Iván Werning, Fiscal Unions, *American Economic Review* 107 (2017), 3788–3834.

[7] Frankel, Jeffrey and Andrew Rose, The Endogeneity of the Optimum Currency Area Criteria, *Economic Journal* 108 (1998), 1009–1025.

[8] Gomis-Porqueras, Pedro, Timothy Kam and Christopher Waller, Nominal Exchange Rate Determinacy under the Threat of Currency Counterfeiting, *American Economic Journal: Macroeconomics* 9 (2017), 256–273.

[9] Grimm, Louisa, Sven Steinkamp, and Frank Westermann, On Optimal Currency Areas: Common Shocks Versus Common Persistence of Shocks, *International Journal of Finance and Economics* (2025), DOI: 10.1002/ijfe.3093.

[10] Hu, Tai-Wei, John Kennan, and Neil Wallace, Coalition-proof Trade and the Friedman Rule in the Lagos-Wright Model, *Journal of Political Economy* 117 (2009), 116-137.

[11] Hu, Tai-Wei and Guillaume Rocheteau, On the Coexistence of Money and Higher-Return assets and its Social Role, *Journal of Economic Theory* 148 (2013), 2520-2560.

[12] Kareken, John and Neil Wallace, On the Indeterminacy of Equilibrium Exchange Rates, *Quarterly Journal of Economics* 96 (1981), 207-222.

[13] Kenen, Peter, The Theory of Optimum Currency Areas: An Eclectic View, In *Monetary Problems of the International Economy*, edited by Robert A. Mundell and Alexander K. Swoboda, 1969, Chicago: University of Chicago Press.

[14] Lagos, Ricardo and Randall Wright, A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy* 113 (2005), 463-484.

[15] Lucas, Robert E., Asset Prices in an Exchange Economy, *Econometrica* 46 (1978), 1429-1445.

[16] McKinnon, Ronald I., Optimum Currency Areas and Key Currencies: Mundell I versus Mundell II, *Journal of Common Market Studies* 42 (2004), 689-715.

[17] Mundell, Robert A., A Theory of Optimum Currency Areas, *American Economic Review* 51 (1961), 657–665.

[18] Mundell, Robert A., Uncommon Arguments for Common Currencies, in H.G. Johnson and A.K. Swoboda, *The Economics of Common Currencies*, 1973, Allen and Unwin, 114–32.

[19] O'Neill, Barry, Dov Samet, Zvi Wiener and Eyal Winter, Bargaining with an Agenda, *Games and Economic Behavior* 48 (2004), 139–153.

[20] Rocheteau, Guillaume and Ed Nosal, Money, Payments, and Liquidity, Second Edition, 2017, The MIT Press.

[21] Rocheteau, Guillaume and Randall Wright, Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium, *Econometrica* 73 (2005), 175-202.

[22] Shapley, Lloyd and Martin Shubik, Trade Using One Commodity as a Means of Payment, *Journal of Political Economy* 85 (1977), 937-968.

[23] Wallace, Neil, The Mechanism-Design Approach to Monetary Theory, *Handbook of Monetary Economics* 3 (2010), 3-23.

[24] Zhang, Cathy, An Information-based Theory of International Currency, *Journal of International Economics* 93 (2014), 286-301.

[25] Zhu, Tao, Equilibrium Concepts in the Large Household Model, *Theoretical Economics* 3 (2008), 258-281.

[26] Zhu, Tao and Neil Wallace, Pairwise Trade and Coexistence of Money and Higher-Return Assets, *Journal of Economic Theory* 133 (2007), 524-535.