Policy Coordination, Monetary Independence, and Optimal Inflation

Zhe Li and Tao Zhu*

January 31, 2025

Abstract

When a fiscal authority (FA) and an independent central bank (CB) cannot commit to any future actions, they select a current policy through a policy-coordination game associated with a bargaining norm. Households vote each period; when voting, they temporally differ in productivity. FA inherits the winning majority's preference. CB cares average welfare. CB's policy instrument is the nominal interest rate; FA's is lump taxes. When CB has some bargaining power, the bargaining outcome lowers inflation than what the majority favors. Determinacy comes by if the bargaining norm specifies the tax response to the future asset value by a suitable manner.

JEL: E5, E6

Keywords: Fiscal policy, Monetary policy, Coordination, Monetary Independence, Bargaining

*Li, School of Economics, Shanghai University of Finance and Economics, 777 Guoding Road, Shanghai 200433, China. Email: li.zhe@mail.shufe.edu.cn. Zhu, Hong Kong University of Science and Technology. Email: taozhu@ust.hk.

1 Introduction

In the modern literature, viewing inflation as a fiscal problem can be at least dated back to Sargent and Wallace [33]. In the context of Sargent and Wallace [33], the selection of the government policy is a coordination game between an independent central bank (CB hereafter) and a fiscal authority (FA hereafter)—CB and FA have to coordinate because there is a consolidated government constraint.[1] Sargent and Wallace [33] seem to work on a version of Wallace's "game of chicken" (see Sargent [32]) and focus on the fiscal-leadership case (i.e., FA moves first to select the fiscal-deficit path). They show that if FA creates a positive present value of deficits, then CB must inflate at some time point to finance the deficits. In the monetary-leadership case (i.e., CB moves first to select the money-growth path), CB can keep inflation under control if it desires.

But why does FA desire to create deficits at the first place? Why does CB desire to control inflation? Suppose FA really desires to create deficits and CB really wants to control, and, suppose, realistically, CB does not have a dominant role in the policy-selection process. Then, would CB and FA reconcile their different interests and, in particular, get inflation tampered down? Or would they fight each other to amplify inflation? On a general level, would there be a mechanism that can anchor the price level and inflation when the decision makers have different interests? Sargent and Wallace [33] are silent about these issues, issues that are better addressed in a model where CB and FA both have welfare-based preferences. In many basic monetary models, the Friedman rule is optimal, i.e., deflation instead of inflation improves welfare. Because the fiscal problems are often associated with politics, we find it suitable for the benchmark purpose to start from a model where the Friedman rule is optimal but not selected because of political reasons. This paper studies such a model.

Our model is a version of the Lagos-Wright [19] model. Each period has three stages of trade between goods and (nominal) assets. The government has exogenously-fixed expenditures in stage-1 goods; old (nominal) bonds mature before stage-1 trade; and taxes are collected and new bonds are issued after. CB's policy instrument is the nominal interest rate and FA's is lump sum taxes/transfers. Newly issued bonds are

[1]See Basseto and Sargent [5] for a recent review of the literature that builds on the fiscal-monetary policy coordination by the government's budget constraint.

illiquid at stage-2 trade (which permits a positive nominal interest rate). Households have the linear preference at stage-3 trade (which keeps the post-trade distribution of wealth degenerate). Idiosyncratic shocks split households into two transient groups before stage-1 trade: one with high productivity in stage-1 trade and another with low productivity. We make three related but distinct contributions against this model.

First, we reveal a realistic political channel by which an explicit welfare-based concern drives inflation. In our model, households vote each period before stage-1 trade. FA's preference is determined by the preference of the winning majority; CB cares average welfare; and neither authority can commit to its future policy-instrument choices. If CB dictates all the current policy, then it follows the Friedman rule. If FA dictates, then it deviates from the Friedman rule when the majority of households have low productivity. The reason is simple—the majority do not fully internalize the benefit of the enhanced asset's value from deflation financed by lump sum taxes and, in the meanwhile, to enjoy that benefit, each household in the majority bears a higher cost than an imaginary average household. So when politics affects policy selection, even transitory heterogeneity can drive inflation.

Second, we formulate the interaction between CB and FA that goes beyond one authority being a dictator or leader. In our model, FA and CB reconcile their interests by a bargaining norm which specifies a current policy efficient from their eyes. Each authority can choose to fight with the other, meaning that CB and FA select values of their own instruments value independently and simultaneously. When the majority of households have low productivity, a higher nominal interest rate and a higher inflation rate (with respect to any efficient policy) reinforce each other in the mutual best response. Because of this, FA and CB follow the bargaining norm to keep the nominal interest low and tamper the inflation down. To our best knowledge, our paper is the first explicit application of the notion of bargaining to the CB-FA interaction for the policy selection.

Third, we find a fiscal mechanism for determinacy when policy makers are not committed to any future actions. Think of an equilibrium when CB has all the bargaining power. Can this equilibrium be a determinant equilibrium? When CB has all the bargaining power, the option of fighting is constraining for PA to participate in bargaining as FA incurs some loss by keeping inflation low. Because each authority's payoff from fighting depends on the asset's future value, how much the current production can be pushed by CB is linked to the future production through the asset's future value. As in a class of monetary models, a link between the current and future production is the potential source of real indeterminacy. Our mechanism for determinacy is fiscal by two reasons. First of all, the mechanism is built on the current tax response to the asset's future value. Because each authority takes the asset's future value as given and understands that the asset's current value is relevant for its objective, it has an incentive to adjust taxes when the asset's future value deviates from the equilibrium path. Secondly, the tax response is in a way that FA has some bargaining power in some circumstance—for the equilibrium in concern, this means that FA has some bargaining power off the equilibrium path.

The related literature

Our paper is related to at least four strands of the literature. The first strand of the literature tackles relevance of independence of CB. This literature is largely built on time inconsistency problems (see Kydland and Prescott [18] and Barro and Gordon [4])—time inconsistency creates an inflation bias so that an independent central banker who dislikes inflation can be a solution to the bias (see, Rogoff [31], Walsh [38], and Persson and Tabellini [29]).[2] Our model abstracts away time inconsistency problems but explores the value of an independent CB in its interaction with FA.

The second strand studies the strategic CB-FA interaction when the two authorities have different objectives. Early contribution includes Tabellini [37], Alesina and Tabellini [1], and Dixit and Lambertini [8]. With enriched macroeconomic details, most recent papers have inflation root in a time-inconsistency problem, and, as the early contribution, focus on either a leadership or simultaneous-move game; see, e.g., Niemann [26], Niemann et al. [27], Miller [24], and Camous and Matveev [7].[3] In one important aspect, the modelling approach of Miller [24] is similar to ours; that is, the political process generates a difference between the objectives of two authorities. In Miller's model, agents are all alike and CB's objective is social welfare; the political process gives some agent a chance to dictate the fiscal actions and, hence, distorts the fiscal component of the policy;[4] and an independent CB reduces the fiscal distortion

[2]Eggertsson and Borgne [10] show that in the absence of any time inconsistency problem, an independent central bank with long-term employment can improve the quality of decision because he is incentivized to exert effort in decision making and is insulated from election pressure.

[3]In a model with a time-inconsistency problem, Martin [22] studies the joint determination of monetary and fiscal policies by a current unified government versus the future government.

[4]Niemann [26] and Niemann et al. [27] assign distinct unility functions to the two authorities. Camous and Matveev [7] let the central bank follow a strategic monetary rule, which targets inflation conditional on fiscal policy.

but because of time inconsistency, it encourages inflation.

The third strand examines inflation from its taxation nature (see Nowotny [14] and Schmitt-Grohe and Uribe [34] for this extensive literature). Methodologywise, one paper close to ours is Albanesi [2]. In Albanesi's model, rich and poor agents directly bargain over the whole policy, no group has an independent policy instrument, and the two groups reconcile because of an exogenously-given breakdown cost. In our model, the policy bargaining is between CB and FA, each authority has its own policy instrument, and the two authorities reconcile because of an endogenous fighting cost.

The fourth strand concerns real and nominal determinacy or indeterminacy under a variety of monetary/fiscal policy rules (see Sims [34] and Grembi et al [14] for this extensive literature). As already noted, our model does not have any monetary/fiscal rule. With some modification, our model can generate certain convenient patterns among the government expenditures, inflation and nominal interest rates. An outsider observer may infer from those patterns that CB and FA follow some policy rules . Our message, however, is that this inference can be misleading because the convenient patterns are just an outcome of the optimal decision making of CB and FA.

2 The model

Time is discrete and infinite, dated as $t \geq 0$. Each period t is divided into three stages, 1, 2, and 3. The economy is populated with a measure one of continuum infinitely lived households. Each household consists of two agents: a consumer and a producer. At stage $n \in \{1, 2\}$ of a period, there are two produced goods, I and II. For one half of households, their consumers can consume good II and producers can produce good I, while for another half, consumers consume good II and producers produce good I. At stage 3, there is one good which can be consumed by all consumers and produced by all producers. All stage goods perish when the corresponding stages end. A household's period utility is

$$u_1(x_1) - \theta c_1(y_1) + u_2(x_2) - c_2(y_2) + x_3 - y_3,$$

where x_n is the the consumer's consumption of (suitable) goods and y_n is the producer's production at stage n, and θ is an idiosyncratic shock that is realized at the start of the period. The shock $\theta \in \{h, l\}$, where $0 < l < h$, is i.i.d. across time and across households. The utility functions u_n is strictly increasing, strictly concave, and twice continuously differentiable with $u_n(0) = 0$, $u'_n(0) = \infty$, and $u'_n(\infty) = 0$, $n = 1, 2$. The disutility function c_n is strictly increasing, weakly convex, and twice continuously differentiable with $c_n(0) = 0$, $n = 1, 2$. The household maximize discount expected utility with the discount factor is $\beta \in (0, 1)$.

There is a government. Among others, the government consumes $0.5g \geq 0$ units of stage-1 goods K$\in$ {I,II}, with g being exogenously set, and as detailed below, issues two nominal assets, money and government bonds.

There are three separated locations: a central location, trading location I, and trading location II. The trading locations accommodate the exchange of goods with nominal assets at each stage. The central location accommodates activities only involving nominal assets at stage 1. Households start each stage from the central location.

When $n \in \{1, 2\}$, consumers who consume good K$\in$ {I,II} and producers who produce good K travel to trading location K from the central location. When $n = 3$, the consumer and producer from the same household randomly select a trading location and together travel there from the central location. In each trading location, all agents are anonymous, ruling out private credits; but there is a competitive market for agents to exchange some nominal assets with goods. A household's nominal earnings from one trading location cannot be transferred to another trading location within a stage. After trading, consumers consume in the trading locations and then all agents return to the central location. Because the two trading locations are symmetric at each stage, we denote by P_t^n the nominal price of goods at the stage-n trading location of period t. The government prints $P_{1t}g$ amount of money to buy goods in the trading locations at stage 1.

After agents return to the central location at stage 1 of period t, the government collects nominal lump taxes $P_{1t}\tau_t$ from households, and issues nominal bonds which mature, i.e., automatically turn into money, at the start of period $t + 1$. Notice that the government effectively injects $P_{1t}(g - \tau_t)$ amount of nominal assets (in the form of money) at stage 1 of period t if $\tau_t \leq g$ or withdraws $P_{1t}(\tau_t - g)$ if $\tau_t > g$.

At the same time as the government issues bonds in the central location, households can also issue private nominal bonds there, which are paid back at the start of period $t + 1$.[5] The government bonds and private bonds are illiquid at stage 2 but

[5] In order for the government to collect lump sum taxes, a household ought to have an identity which is verfiable in the central location. This identify permits the private borrowing and lending in

liquid at stage 3.[6] Because private and government bonds are perfect substitutes for a holder, they share the same nominal interest rate.

The government operates in the central location a discount window which exchanges money with private and government bonds by a fixed nominal interest, denoted i_t (the discount window can be replaced with open market operation). No free arbitrage implies that i_t is also the nominal interest rate on the bonds market. After the government closes the discount window, there is no other activity at stage 1.

We describe what constitutes a policy in section 3 and how a policy is selected by a coordination game between a fiscal authority and a monetary authority in section 5. Our key modeling choices in this section are all related to the section-5 coordination game, and shall be discussed at the end of section 5. It suffices to note here that all ingredients of our model are standard.

3 Market equilibrium

In each period t, households take the current and future prices of goods and bonds $\{P_{1s}, P_{2s}, P_{3s}, i_s\}_{s\geq t}$ and current and future lump sum taxes $\{\tau_s\}_{s\geq t}$ as given. A market equilibrium is attained when the optimal choices of households clear all markets.

Period-t household constraints and government budget constraint

For a household that enters period t with nominal assets z_{t-1} and receives the shock θ, let $x_{\theta nt}$ denote its consumer's stage-n consumption and and $y_{\theta nt}$ its producer's stage-n production, $m_{\theta nt}$ denote the amount of money carried by the household into stage $n \in \{2, 3\}$, $b_{\theta t}$ denote the amount of (private and government) bonds owned by the household at the end of stage 1, and z_t denote the amount of nominal assets owned by the household at the end of period t. Then the household is subject to the constraints

$$P_{1t}x_{\theta 1t} \leq z_{t-1},\ \ P_{2t}x_{\theta 2t} \leq m_{\theta 2t}, \tag{1}$$

$m_{\theta 2t} + b_{\theta t}(1 + i_t)^{-1} = z_{t-1} + P_{1t}(y_{\theta 1t} - x_{\theta 1t} - \tau_t)$, $m_{\theta 3t} - m_{\theta 2t} = P_{2t}(y_{\theta 2t} - x_{\theta 2t})$, and $z_t = m_{\theta 3t} + b_{\theta t} + P_{3t}(y_{\theta 2t} - x_{\theta 3t})$. The two constraints in (1) present because the

the central location. It does not support private credits in the trading locations as it is not verifable there.

[6]One may assume that government bonds are nontransferable book entry at stage 2 and are in some transferable and verifiable form at stage 3. We may assume that private bonds are always illiquid (this can be justified by assuming that private bonds can be counterfeited) and our result carries over with completely illiquid private bonds.

nominal earnings in one location cannot be transferred to another within a stage, and money is the only nominal asset in the second constraint because bonds are illiquid at stage 2.

Let M_t denote the amount of money and B_t denote the amount of government bonds held by households at the end of stage 1 of period t—a negative B_t means that the government lends money to agents by the discount window. Let $Z_t = M_t + B_t$, which is the total amount of nominal assets held by households at the start of period $t+1$. Each household holds a pre-determined stock Z_0 of nominal assets at the start of period 0. The government budget constraint can be expressed as $Z_{t-1} + P_{1t}(g - \tau_t) = M_t + B_t/(1 + i_t)$ or, more familiarly, as

$$P_{1t}(g - \tau_t) = M_t + B_t/(1 + i_t) - M_{t-1} - B_{t-1}. \tag{2}$$

Period-t household optimal choices

Because of the linear preference of the stage-3 goods, in any equilibrium, when the household enters stage 3 of period t with z units of nominal assets, its continuation value is $\Pi_t(z) = z/P_{3t} + C_t$ for some constant C_t; moreover, when all households enter period t with Z_{t-1} units of nominal assets, they all leave with Z_t. The affine continuation value function Π_t implies the following standard sufficient and necessary optimality conditions for $z_{t-1} = Z_{t-1}$,

$$c_2'(y_{\theta 2t}) = \frac{P_{2t}}{P_{3t}},\ \theta c_1'(y_{\theta 1t}) = \frac{u_2'(x_{\theta 2t})P_{1t}}{P_{2t}}, \theta c_1'(y_{\theta 1t}) = \frac{(1 + i_t)P_{1t}}{P_{3t}}, \tag{3}$$

$$u_1'(x_{\theta 1t}) \geq \theta c_1'(y_{\theta 1t}), x_{\theta 1t} < Z_{t-1}/P_{1t} \text{ only if } u_1'(x_{\theta 1t}) = \theta c_1'(y_{\theta 1t}). \tag{4}$$

The first condition in (3) says that the marginal cost of obtaining one unit of money from stage 2 is equal to the marginal return of the money carried to stage 3, the second says that the marginal cost of obtaining one unit of money from stage 1 is equal to the marginal utility brought by one unit of money at stage 2, and the third says that the marginal cost of obtaining one unit of bonds from stage 1 is equal to the marginal return of the bond carried to stage 3. The condition in (4) says that the first constraint in (1) is not binding only if the marginal utility of consumption is equal to the marginal distuility of production at stage 1.

By (4), the household's optimal $x_{\theta 1t}$ is either $x_{\theta 1t} = Z_{t-1}/P_{1t}$ all θ or $x_{\theta 1t} < Z_{t-1}/P_{1t}$ and $u_1'(x_{\theta 1t}) = \theta c_1'(y_{\theta 1t})$ some θ. Because $\theta c_1'(y_{\theta 1t})$ does not depend on θ, in the latter case, $x_{\theta 1t}$ does not depend on θ and $u_1'(x_{\theta 1t}) = \theta c_1'(y_{\theta 1t})$ all θ. Moreover, in the latter case, $x_{1t} = Z_{t-1}/P'_{1t}$ for some $P'_{1t} < P_{1t}$. As it becomes clear soon, the value of P_{1t} is affected by the policy. For our purpose, it is without loss of generality to concentrate on the value of P_{1t} (or equivalently the underlying policy) such that the consumer of the household spends Z_{t-1} in the stage-1 trading market. The first condition in (3) implies that the household's optimal $x_{\theta 2t}$ and $y_{\theta 2t}$ do not depend on the household's type θ. If $i_t > 0$ then the household's optimal $m_{\theta 2t}$ is equal to $m_{\theta 2t} = P_{2t}x_{2t}$; if $i_t = 0$, it is without loss of generality to assume that $m_{\theta 2t}$ is just sufficient to buy x_2 at stage 2. Thus we write the household's optimal $(x_{\theta 1t}, y_{\theta 1t}, x_{\theta 2t}, y_{\theta 2t}, m_{\theta 2t})_\theta$ as $(x_{1t}, y_{h1t}, y_{l1t}, x_{2t}, y_{2t}, m_{2t})$ with

$$x_{1t} = \frac{Z_{t-1}}{P_{1t}},\ u'_1(x_{1t}) \geq \theta c'_1(y_{\theta 1t}),\ x_{2t} = \frac{m_{2t}}{P_{2t}}. \tag{5}$$

It is convenient to describe conditions for $(x_{1t}, y_{h1t}, y_{l1t}, x_{2t}, y_{2t}, m_{2t})$ by normalized terms. Let $L_t = M_t + B_t/(1 + i_t)$ and

$$\phi_{1t} = \frac{L_t}{P_{1t}},\ \phi_{2t} = \frac{Z_t}{P_{2t}}, \phi_{3t} = \frac{Z_t}{P_{3t}},\ \delta_t = \frac{Z_{t-1} - L_t}{Z_{t-1}},\ \lambda_t = \frac{M_t}{L_t}; \tag{6}$$

then conditions in (3) and (5) can be written as

$$c'_2(y_{2t}) = \frac{\phi_{3t}}{\phi_{2t}},\ \theta c'_1(y_{\theta 1t}) = \frac{\phi_{2t}}{\phi_{1t}} \frac{u'_2(x_{2t})}{1 + (1 - \lambda_t)\, i_t},\ \theta c'_1(y_{\theta 1t}) = \frac{\phi_{3t}}{\phi_{1t}} \frac{1 + i_t}{1 + (1 - \lambda_t)\, i_t}, \tag{7}$$

$$x_{1t} = \frac{\phi_{1t}}{1 - \delta_t},\ u'_1(x_{1t}) \geq \theta c'_1(y_{\theta 1t}),\ x_{2t} = \frac{\lambda_t \phi_{2t}}{1 + (1 - \lambda_t)\, i_t}. \tag{8}$$

For the normalized terms, one may think that new bonds are issued after money is withdrawn by lump sum taxes at stage 1 so that L_t is the amount of assets right before bond issuance in period t, ϕ_{1t} is the real value of pre-bond-issuance assets measured by the stage-1 price, ϕ_{nt} is the real value of post-bond-issuance assets measured by the stage-2 price for $n \in \{2, 3\}$, δ_t is the rate of assets withdrawn from the economy before bond issuance, and λ_t is the proportion of assets carried in the form of money after bond issuance . Note that λ_t can be greater than unity as B_t can be negative.

Government policy and equilibrium

Using (6) and the first quality in (8), we can write the government budget constraint (2) as

$$\delta_t x_{1t} = \tau_t - g. \tag{9}$$

With g fixed in (9), only one of δ_t or τ_t can be a *free* policy choice. Also, only one of λ_t

or i_t can be a free policy choice (i.e., either B_t is fixed by policy and i_t endogenously responds to clear the bond market, or i_t is fixed by policy and B_t endogenously responds). We use τ_t and i_t as the free policy choices as they seem more conventional. We refer to $\zeta_t \equiv (\tau_t, i_t)$ as a *temporary policy* in period t, i_t as a *monetary component* of the temporary policy, τ_t as the *fiscal component* of the policy, and a sequence $\{\zeta_t\}_{t\geq 0}$ of temporary policies as a *policy*.

We refer to $\phi_t \equiv (\phi_{1t}, \phi_{2t}, \phi_{3t})$ as a *market price vector* in period t. Given a temporary policy ζ_t in period t, we refer to a market price vector when an exogenously given price υ is treated as the price ϕ_{3t} as a temporary market equilibrium if the price vector clears the markets in period t. Using (6), we can write the period-t market-clearing conditions on goods as

$$x_{1t} = \textstyle\sum_{\theta} \pi_{\theta} y_{\theta 1t} - g \text{ and} x_{2t} = y_{2t}. \tag{10}$$

Definition 1 *A market price vector ϕ_t is a temporary market equilibrium admitted by a price $\upsilon > 0$ and a temporary policy ζ_t if there exists a tuple $\varsigma_t \equiv (x_{1t}, y_{h1t}, y_{l1t}, x_{2t}, y_{2t}, \delta_t, \lambda_t)$ satisfying (7)-(10) when $\phi_{3t} = \upsilon$.*

Given a policy, a market equilibrium endogenizes all prices by the market-clearing conditions. To relate the price vector in period t to the price vector and the temporary price in period $t+1$, let us consider a household whose period-t optimal choices satisfy (7)-(10). Anticipating to spend Z_t at the stage-1 trading market in period $t+1$, the necessary and sufficient condition for the household to carry Z_t into period $t+1$ is

$$\phi_{3t} = \beta \frac{\phi_{t+1}}{1-\delta_{t+1}} u_1'\left(\frac{\phi_{t+1}}{1-\delta_{t+1}}\right), \tag{11}$$

saying that the marginal cost of obtaining one unit of money or bond from current stage 3 is equal to the discounted marginal return of the money carried to stage 1 in the next period.

Definition 2 *A sequence of market price vectors $\{\phi_t\}_{t\geq 0}$ is a market equilibrium admitted by a policy $\{\zeta_t\}_{t\geq 0}$ if for each t, ϕ_t is a temporary market equilibrium admitted by (υ, ζ_t) when $\upsilon = \phi_{3t}$ and (11) holds.*

4 Equilibrium allocation and *ex ante* optimal policy

Here we first characterize a temporary market equilibrium and a market equilibrium in terms of a temporary allocation and an allocation.

Lemma 1 *Fix $\upsilon > 0$ and let $\Xi(\upsilon, t) = \{\zeta_t = (\tau_t, i_t) : \upsilon$ and ζ_t admit a temporary market equilibrium$\}$. Fix $\zeta_t \in \Xi(\upsilon, t)$. Then υ and ζ_t admit a unique tuple ς_t, in which the triplet $(y_{h1t}, y_{l1t}, y_{2t})$, referred to the temporary allocation supported by (υ_t, ζ_t), satisfying*

$$u_2'(y_{2t}) = (1 + i_t)c_2'(y_{2t}), \tag{12}$$

$$\theta c_1'(y_{\theta 1t}) = \frac{\upsilon + i_t y_{2t} c_2'(y_{2t})}{x_{1t}(1 - \delta_t)}, \tag{13}$$

and $u_1'(x_{1t}) \geq \theta c_1'(y_{\theta 1t})$, where $x_{1t} = \sum_\theta \pi_\theta y_{\theta 1t} - g$ and $\delta_t = (\tau_t - g)x_{1t}^{-1}$; and in this ς_t, $\lambda_t = (1 + i_t)y_{2t}c_2'(y_{2t})[\upsilon_t + i_t y_{2t} c_2'(y_{2t})]^{-1}$.

Proof. We delegate proof not present in the main text to the appendix. ■

Lemma 2 *Suppose a policy $\{\zeta_t\}_{t\geq 0}$ admits an equilibrium $\{\phi_t\}_{t\geq 0}$. Then there exists a unique sequence of $\{(y_{h1t}, y_{l1t}, y_{2t})\}_{t\geq 0}$, referred to an allocation supported by $\{(\zeta_t, \phi_t)\}_{t\geq 0}$, such that $(y_{h1t}, y_{l1t}, y_{2t})$ satisfies the conditions in Lemma 1 when υ in (13) is replaced with ϕ_{3t} in (11) all t.*

Now, as a reference point, consider the scenario that a social planner picks a policy at the start of period 0 before agents know their types. Because agents are alike at this time, the planner is to maximize the representative agent's expected discount utility. Because of the linear preference of stage-3 goods, the representative household's expected discount utility in an equilibrium $\{\phi_t\}_{t\geq 0}$ admitted by policy $\{\zeta_t\}_{t\geq 0}$ is completely determined by the allocation $\{(y_{h1t}, y_{l1t}, y_{2t})\}_{t\geq 0}$ supported by $\{(\zeta_t, \phi_t)\}_{t\geq 0}$ as

$$\sum\nolimits_{t\geq 0} \beta^t [\mu_1(y_{h1t}, y_{l1t}) + \mu_2(y_{2t})],$$

where $\mu_1(y_{h1}, y_{l1}) = u_1(\sum_\theta \pi_\theta y_{\theta 1} - g) - \sum_\theta \pi_\theta c_1(y_{\theta 1})$ and $\mu_2(y_2) = u_2(y_2) - c_2(y_2)$. Let $\Gamma(\{\zeta_t\})$ be the maximum of the representative agent's expected discount utility over the set of allocations, each allocation of which is supported by (ζ_t, ϕ_t) for some ϕ_t

admitted by $\{\zeta_t\}$. Then a policy $\{\zeta_t\}$ is *ex ante* optimal if $\{\zeta_t\} \in \arg\max_{\{\zeta'_t\}} \Gamma(\{\zeta'_t\})$, and the corresponding allocation is *ex ante* optimal if ζ is *ex ante* optimal.

Lemma 3 *Given $y_{h1} \geq 0$, let $y_{l1}(y_{h1})$ denote the unique value of y_{l1} satisfying $hc'_1(y_{h1}) = lc'_1(y_{l1})$. Let*

$$(y^*_{h1}, y^*_{l1}) = \arg \max_{(y_{h1}, y_{l1}) \geq (0,0)} \mu_1(y_{h1}, y_{l1}) \text{ s.t. } y_{1l} = y_{l1}(y_{h1}). \tag{14}$$

Let y^*_2 satisfy $\mu'_2(y^*_2) = 0$. Let $\{(y_{h1t}, y_{l1t}, y_{2t})\}_{t\geq 0}$ have $(y_{h1t}, y_{l1t}, y_{2t}) = (y^*_{h1}, y^*_{l1}, y^*_2)$ all t. Then the Friedman rule, i.e., the policy $\{\zeta_t\}_{t\geq 0}$ with $\tau_t = (1-\beta)\sum_\theta \pi_\theta y^*_{\theta 1} + \beta g$ and $i_t = 0$ all t, is ex ante optimal and admits $\{(y_{h1t}, y_{l1t}, y_{2t})\}_{t\geq 0}$ as the ex ante optimal allocation.

5 Policy coordination

Departing from the section-4 reference point, let us consider that the government consists of two independent entities, FA which controls the fiscal component τ_t of a temporary policy and CB which controls the monetary component i_t in each period t. In period t, there is voting right after households know their types. Every household has one vote and FA's objective is to maximize welfare of the winning majority. CB's objective is to maximize average welfare of all households. Neither authority can commit to its future policy selection.

The policy selection is done at stage 1 before agents visit the trading locations through a coordination game between CB and FA. Associated with a bargaining norm, the coordination game has two rounds of moves. A *bargaining norm* is a mapping $(\upsilon, t) \mapsto \zeta(\upsilon, t)$ which specifies for $\upsilon > 0$ a *reconciliation temporary policy* $\zeta(\upsilon, t)$ in period t. At the first round of the game, FA and CB simultaneously say yes or no. If one authority says no, then CB selects i_t and FA selects τ_t simultaneously and independently, and the game ends after the selections are made; otherwise, the second round starts. At the second round, CB proposes ζ_t and FA says yes or no. If yes, then ζ_t is the selection outcome; otherwise, $\zeta(\upsilon, t)$ is.

To formally describe each authority's valuation of a policy in the game, let us refer to Lemma 1. Taking $\phi_{3t} = \upsilon > 0$ as given, the two authorities understand that they must select a temporary policy $\zeta_t \in \Xi(\upsilon, t)$ so that the households can end up with a temporary market equilibrium in that period.

Lemma 4 *Fix $\upsilon > 0$ and $\zeta_t \in \Xi(\upsilon, t)$. Let $(y_{h1t}, y_{l1t}, y_{2t})$ be the temporary allocation supported by (υ, ζ_t), $\hat{\theta}$ be the majority type of households, and*

$$\eta(y_{h1}, y_{l1}, h) = \sum\nolimits_{\theta} u_1(\pi_\theta y_{\theta 1} - g) - hc_1(y_{h1}) - \pi_l(y_{l1} - y_{h1})hc_1'(y_{h1}). \qquad (15)$$

Then the values or payoffs of ζ_t for a type-θ household, for FA, and CB are $V(\zeta_t, \upsilon, \theta) \equiv \eta(y_{h1t}, y_{l1t}, \theta) + \mu_2(y_{2t})$, $V(\zeta_t, \upsilon, \hat{\theta})$, and $W(\zeta_t, \upsilon) \equiv \sum_\theta \pi_\theta V(\zeta_t, \upsilon, \theta) = \mu_1(y_{h1t}, y_{l1t}) + \mu_2(y_{2t})$, respectively.

In playing the coordination game, if one authority says no at the first round, then in equilibrium each authority's selection must be a best response to another. This mutual best response outcome, denoted by $\zeta^\circ(\upsilon, t) = (i^\circ(\upsilon, t), \delta^\circ(\upsilon, t))$ and referred to as the *fighting policy*, satisfies

$$\tau^\circ(\upsilon, t) \in \arg\max_{\tau_t} V((\tau_t, i^\circ(\upsilon, t)), \upsilon, \hat{\theta}_t) \text{ and } i^\circ(\upsilon, t) \in \arg\max_{i_t} W((\tau^\circ(\upsilon, t), i_t), \upsilon). \qquad (16)$$

To ensure existence of a mutual best response, we maintain the following conditions on the function c_1.

Condition 1 *(a) $z \mapsto c_1'(z)$ is weakly convex; and (b) for any given positive scalar a, $z \mapsto c_1'^{-1}(ac_1'(z))$ is weakly convex.*

We require that a bargaining norm $\zeta(.)$ satisfies $\zeta(\upsilon, t) \in \Xi^e(\upsilon, t)$ all (υ, t), where

$$\Xi^e(\upsilon, t) = \{\zeta_t : \zeta_t \in \arg\max_{\zeta_t' \in \Xi(\upsilon, t)} W(\zeta_t', \upsilon) \text{ s.t. } V(\zeta_t, \upsilon, \hat{\theta}) \geq C,\, C \geq V(\zeta^\circ(\upsilon, t), \upsilon, \hat{\theta})\};$$

that is, the reconciliation temporary policy $\zeta(\upsilon, t)$ assigned by the bargaining norm $\zeta(.)$ is efficient from the perspective of CB and FA. As such, selecting $\zeta(\upsilon, t)$ is the equilibrium outcome of the policy-coordination game.

Definition 3 *Given a bargaining norm $\zeta(.)$, a sequence $\{(\zeta_t, \phi_t)\}_{t \geq 0}$ is a political-economy equilibrium if for each t, $\zeta_t = \zeta(\upsilon, t)$ and ϕ_t is a temporary market equilibrium admitted by (υ, ζ_t) when $\upsilon = \phi_{3t}$ and (11) holds.*

We now relate our key modeling choices in section 2 to the policy-coordination game. We use the consumer-producer household structure to simplify the composition of voters so that voters only differ in one dimension at the voting time and there are only two groups of voters. We use stage-1 and stage-2 trades to ensure that CB's pick of the nominal interest rate is independent of FA's pick of lump sum taxes. We may alternatively follow Willamson [39] to have this independence; that is, replace stage 1 with stage 3, have only two stages of trade, and let bonds be partially illiquid at stage 2.[7] Because we need idiosyncratic shocks at stage 1, this alternative approach does not seem to bring in additional analytical convenience.

6 Results

Here we first present a partial-equilibrium result, which characterizes $\Xi^e(\upsilon, t)$ for an arbitrary $\upsilon > 0$ and the corresponding set of temporary allocations, i.e.,

$$Y^e(\upsilon, t) = \{(y_{h1t}, y_{l1t}, y_{2t}) : (y_{h1t}, y_{l1t}, y_{2t})\text{supported by } (\upsilon, \zeta_t) \text{ for some } \zeta_t \in \Xi^e(\upsilon, t)\};$$

and, next, using this partial-equilibrium result, we establish a couple of general-equilibrium results when the only exogenous object is a given bargaining rule. For all these results, it is convenient to introduce a familiar bargaining norm, namely, a generalized Nash bargaining norm, which assigns to a given price υ the policy

$$\zeta^{\varphi(\upsilon,t)} \max_{\zeta_t \in \Xi(\upsilon,t)} [W(\zeta_t, \upsilon) - W(\zeta^\circ(\upsilon, t), \upsilon)]^{\varphi(\upsilon,t)} [V(\zeta_t, \upsilon, \hat{\theta}) - V(\zeta_t^\circ(\upsilon), \upsilon, \hat{\theta})]^{1-\varphi(\upsilon,t)}, \quad (17)$$

where $\varphi(\upsilon, t) \in [0, 1]$ is the bargaining power of CB. We denote by $\bar{\zeta}(.)$ and $\hat{\zeta}(.)$ the norms with $\varphi(\upsilon, t) = 1$ all (υ, t) (CB always has all the bargaining power) and $\varphi(\upsilon, t) = 0$ all (υ, t) (FA always has all the bargaining power) respectively, i.e.,

$$\bar{\zeta}(\upsilon, t) = (\bar{\tau}(\upsilon, t), \bar{\imath}(\upsilon, t)) \in \arg \max_{\zeta_t \in \Xi^e(\upsilon,t)} W(\zeta_t, \upsilon), \quad (18)$$

$$\hat{\zeta}(\upsilon, t) = (\hat{\tau}(\upsilon, t), \hat{\imath}(\upsilon, t)) \in \arg \max_{\zeta_t \in \Xi^e(\upsilon,t)} V(\zeta_t, \upsilon, \hat{\theta}), \quad (19)$$

and refer to $\bar{\zeta}(.)$ and $\hat{\zeta}(.)$ as monetary-dominance and fiscal-dominance norms.

For the partial-equilibrium result, fix $\upsilon > 0$ and we begin with the temporary allocations supported by $(\upsilon, \bar{\zeta}(\upsilon, t))$ and $(\upsilon, \zeta^\circ(\upsilon, t))$ ($\zeta^\circ(\upsilon, t)$ is the fighting policy). Using the equilibrium condition 12, one shall see that $\hat{\imath}(\upsilon, t) = 0$ and y_2^* is the stage-2 production in the temporary allocation supported by $(\upsilon, \hat{\zeta}(\upsilon, t))$. A key observation

[7] A key idea Williamson is that the combination of money and bonds can influence the nominal interest rate. Such an idea also appears in the related studies; see, e.g., Herrenbrueck [16] and Rocheteau et al. [30].

for the subsequent analysis is that the temporary allocations supported by $(\upsilon, \hat{\zeta}(\upsilon, t))$ and $(\upsilon, \zeta^{\circ}(\upsilon, t))$ have the same stage-1 production, which is determined by

$$(\hat{y}_{h1}, \hat{y}_{l1}) = \arg \max_{(y_{h1}, y_{l1}) \leq (y^*_{h1}, y^*_{l1})} \eta(y_{h1}, y_{l1}, \hat{\theta}) \ s.t. \ y_{l1} = y_{l1}(y_{h1}); \tag{20}$$

the optimal solution to the problem in (20) is unique because by Condition 1, $y_{h1} \mapsto \eta(y_{h1}, y_{l1}(y_{h1}), \hat{\theta})$ is strictly concave. For this key observation, first consider that the coordination game ends up with $\zeta^{\circ}(\upsilon, t)$. That being the case, the stage-2 production is pinned by CB's choice $i^{\circ}(\upsilon, t)$. With i_t fixed at $i^{\circ}(\upsilon, t)$, the valuation of a policy in Lemma 4 implies that $\tau^{\circ}(\upsilon, t)$, the best response of FA to $i^{\circ}(\upsilon, t)$ must have $\hat{y}_{\theta 1}$ as the stage-1 production of per type-θ household. Next consider that the bargaining norm is the fiscal-dominance norm and the game ends up with $\hat{\zeta}(\upsilon, t)$. As $\hat{\imath}(\upsilon, t)$ is a fixed number (namely zero), again, FA must have $\hat{y}_{\theta 1}$ as the stage-1 production of per type-θ household.

To say more about $\hat{y}_{\theta 1}$, let us compare the maximization problems in (14) and (20). By that comparison, in order enjoy the same consumption benefit from an increase in the purchasing power of the nominal asset in the stage-1 market due to retiring nominal assets, a high-cost household not only bears a higher production cost than the average household in the stage-1 market but also bears a higher debt burden which affects its net consumption in the stage-3 market. As such, the former household ought to prefer a lower level of stage-1 consumption than the latter. Thus, when $\hat{\theta} = h$, $\hat{y}_{\theta 1} < y^*_{\theta 1}$ for each θ. Indeed, when $\hat{\theta} = h$, the first order condition on $\hat{y}_{h1}$ from the problem in (20) is

$$[\pi_h/\pi_l + y'_{l1}(\hat{y}_{h1})][u'_1(\hat{x}_1) - hc'_1(\hat{y}_{h1})] - [y_{l1}(\hat{y}_{h1}) - \hat{y}_{h1}]hc''_1(\hat{y}_{h1}) = 0, \tag{21}$$

where $\hat{x}_1 = \sum \pi_\theta \hat{y}_{\theta 1} - g$, and the first order condition on y^*_{h1} from the problem in (14) is

$$u'_1(x^*_1) - hc'_1(y^*_{h1}) = 0,$$

where $x^*_1 = \sum \pi_\theta y^*_{\theta 1} - g$. By the same line of reasoning, when $\hat{\theta} = l$, $\hat{y}_{\theta 1}$ must exceed $y^*_{\theta 1}$ if the maximization problem in (20) does not bound $y_{\theta 1}$ by $y^*_{\theta 1}$. The upper bound on $y_{\theta 1}$ is imposed to ensure that FA's choice obeys the market equilibrium condition $u'_1(x_1) \geq \theta c'_1(y_{\theta 1})$ (see Lemma 1). That market equilibrium condition pertains to an individual type-θ household's decision problem on the market, which differs from the policy-selection decision problem of FA, an entity that represents the type-$\hat{\theta}$

households as a whole. We group the analysis so far as follows.

Lemma 5 *Fix* $\upsilon > 0$. *Then* $(\hat{y}_{h1}, \hat{y}_{l1})$ *is the stage-1 production in the temporary allocation supported by* $(\upsilon, \zeta_t^\circ(\upsilon))$ *and by* $(\upsilon, \hat{\zeta}_t(\upsilon))$. *Moreover,* $\hat{y}_{\theta 1} < y^*_{\theta 1}$ *when* $\hat{\theta} = h$, *and* $\hat{y}_{\theta 1} = y^*_{\theta 1}$ *when* $\hat{\theta} = l$.

Our next step is to pin down the stage-2 production in the allocation supported by $(\upsilon, \zeta^\circ(\upsilon, t))$. The equilibrium condition (12) implies that when the nominal interest rate is i, the stage-2 production, denoted $y_2(i)$, is determined by

$$(1+i)c_2'(y_2(i)) = u_2'(y_2(i)), \tag{22}$$

Using (13) and the fact that $(\hat{y}_{h1}, \hat{y}_{l1})$ is the stage-1 production in the temporary allocation supported by $(\upsilon, \zeta^\circ(\upsilon, t))$, we have

$$\hat{x}_1 \theta c_1'(\hat{y}_{\theta 1})(1 - \delta^\circ(\upsilon, t)) = \upsilon + i^\circ(\upsilon, t) y_2(i^\circ(\upsilon, t)) c_2'(y_2(i^\circ(\upsilon, t))), \tag{23}$$

where $\delta^\circ(\upsilon, t) = [\tau^\circ(\upsilon, t) - g](\pi_\theta \hat{y}_{\theta 1} - g)^{-1}$. By (13) and (23), when CB picks $i_t = i$ and FA picks $\tau_t = \tau^\circ(\upsilon, t)$, the stage-1 production of per type-θ household denoted $y_{\theta 1 t}(i; \upsilon)$, is uniquely determined by

$$x_{1t}(i; \upsilon)\theta c_1'(y_{\theta 1}(i)) = \hat{x}_{1t}\theta c_1'(\hat{y}_{\theta 1}) + \frac{i y_2(i) c_2'(y_2(i)) - i^\circ(\upsilon, t) y_2^\circ c_2'(y_2^\circ)}{1 - \delta^\circ(\upsilon, t)}, \tag{24}$$

where $x_1(i; \upsilon) = \sum \pi_\theta y_{\theta 1 t}(i; \upsilon) - g$ and $y_2^\circ = y_2(i^\circ(\upsilon, t))$. Therefore, CB's best response to $\tau^\circ(\upsilon, t)$ must be an i that maximize $\mu_1(y_{h1}(i; \upsilon), y_{l1}(i; \upsilon)) + \mu_2(y_2(i))$, i.e.,

$$i^\circ(\upsilon, t) \in \arg\max_{i \geq 0} [\mu_1(y_{h1}(i; \upsilon), y_{l1}(i; \upsilon)) + \mu_2(y_2(i))]. \tag{25}$$

As it turns out, (23) and (25) imply a condition for $i^\circ(\upsilon, t)$ in terms of $y_2(i^\circ(\upsilon, t))$ and

$$A = \hat{x}_1 \sum \pi_\theta [u_1'(\hat{x}_1) - c_1'(\hat{y}_{\theta 1})][\pi_h + \frac{\pi_l c_1''(\hat{y}_{h1}) c_1'(\hat{y}_{l1})}{c_1'(\hat{y}_{h1}) c_1''(\hat{y}_{l1})} + \frac{\hat{x}_1 c_1''(\hat{y}_{h1})}{c_1'(\hat{y}_{h1})}]^{-1}. \tag{26}$$

Lemma 6 *Fix* $\upsilon > 0$. *Then stage-2 production in the temporary allocation supported by* $(\upsilon, \zeta^\circ(\upsilon, t))$ *is* $y_2(i^\circ(\upsilon, t))$, *and* $i^\circ(\upsilon, t)$ *is an* i *satisfying*

$$(\upsilon + A)[u_2'(y_2) - c_2'(y_2)] = -y_2[u_2'(y_2) - c_2'(y_2)]^2 - A y_2 [u_2''(y_2) - c_2''(y_2)], \tag{27}$$

where $y_2 = y_2(i^\circ(\upsilon, t))$. *In particular,* $i^\circ(\upsilon, t) = 0$ *and* $y_2(i^\circ(\upsilon, t)) = y_2^*$ *when* $\hat{\theta} = l$, *and* $i^\circ(\upsilon, t) > 0$ *and* $y_2(i^\circ(\upsilon, t)) < y_2^*$ *when* $\hat{\theta} = h$.

It may help thinking about the positive nominal interest rate $i_t^\circ(\upsilon)$ when $\hat{\theta}_t = h$ from a slightly different perspective. Consider what CB is to do if it is allowed to reset the nominal interest rate from the fiscal-dominance level $\hat{\tau}_t(\upsilon) = 0$ when δ_t is fixed at the fiscal-dominance level $\hat{\delta}(\upsilon, t)$. By the equilibrium condition (12), a marginal increase in i_t above zero incurs a zero marginal change in stage-2 welfare. The equilibrium condition (13) can be written as $x_{1t}\theta c_1'(y_{\theta 1t}) = R_t$, where

$$R_t = \frac{\upsilon + i_t y_{2t} c_2'(y_{2t})}{1 - \delta_t}; \tag{28}$$

that is, the stage-1 production of each type is increasing in R_t. Because δ_t is fixed, the marginal increase in i_t leads to a marginal increase in the stage-1 production and average stage-1 welfare. Hence, for $\zeta^\circ(\upsilon, t)$ to maintain the same stage-1 production as $\hat{\zeta}(\upsilon, t)$, $\zeta^\circ(\upsilon, t)$ must have the same R_t as $\hat{\zeta}(\upsilon, t)$ but a higher δ_t and a higher i_t than $\hat{\delta}(\upsilon, t)$. In other words, because FA's pick of δ_t in $\zeta^\circ(\upsilon, t)$ keeps the stage-1 production low, there is room for CB to improve its payoff by increasing the stage-1 production if CB's pick of i_t in $\zeta^\circ(\upsilon, t)$ is low, implying that CB's pick of i_t must be high enough to ensure that no authority deviates from $\zeta^\circ(\upsilon, t)$. Everything else equal, a smaller δ_t corresponds to a higher inflation rate in period t. So, simply put, a higher level of inflation and a higher nominal interest rate reinforce each in $\zeta^\circ(\upsilon, t)$ (with respect to $\hat{\zeta}(\upsilon, t)$).

Because $\zeta^\circ(\upsilon, t)$ is dominated by $\hat{\zeta}(\upsilon, t)$ when $i^\circ(\upsilon, t) > 0$ from the perspective of CB and FA, CB and FA are willing to reconcile by the bargaining norm. That is, a positive nominal interest rate that can be carried out independently by CB is the disciplining device that incentivizes the two authorities to reconcile their interests by following the bargaining norm. In reconciliation, $i_t = 0$ (which by (12) implies $y_{2t} = y_2^*$) is the common interest of CB and FA. What each side has to compromise is the fiscal component of the temporary policy—a larger τ_t (equivalent to a larger δ_t) corresponds to a higher stage-1 production and, hence, a smaller gain from reconciliation for FA. When the bargaining norm is the fiscal-dominance norm $\hat{\zeta}(.)$, FA's gain is

$$\Delta(\upsilon) \equiv \mu_2(y_2^*) - \mu_2(y_2(i^\circ(\upsilon, t))), \tag{29}$$

which sets the least upper bound on how much the stage-1 production can be increased from $\hat{y}_{1\theta}$ under the monetary-dominance norm $\bar{\zeta}(.)$. This is the base of our partial-equilibrium result for $\Xi^e(\upsilon, t)$ and $Y^e(\upsilon, t)$.

Proposition 1 *Fix $\upsilon > 0$. Let $\delta(y_{h1}) = \eta(\hat{y}_{h1}, \hat{y}_{l1}) - \eta(y_{h1}, y_{l1}(y_{h1}))$, $Y^e_{h1}(\upsilon) = \{y_{h1} : \delta(y_{h1}) \le \Delta(\upsilon),\ y_{h1} \ge \hat{y}_{h1}\}$, and $\bar{y}_{h1}(\upsilon) = \max Y^e_{h1}(\upsilon)$. Let $\rho(y_{h1}, \upsilon) = \pi_h y_{h1} + \pi_l y_{l1}(y_{h1}) - \upsilon[hc'_1(y_{h1})]^{-1}$. Then $\Xi^e(\upsilon, t) = \{(\tau_t, i_t) = (\rho(y_{h1}, \upsilon), 0) : y_{h1} \in Y^e_{h1}(\upsilon)\}$, and*

$$Y^e(\upsilon, t) = \{(y_{h1}, y_{l1}, y_2) : \hat{y}_{h1} \le y_{h1} \le \bar{y}_{h1}(\upsilon),\ y_{l1} = y_{l1}(y_{h1}),\ y_2 = y_2^*\}.$$

Moreover, for $y_{h1} \in Y^e_{h1}(\upsilon)$, there exists a value of $\varphi(\upsilon, t)$, denoted $\omega(y_{h1}, \upsilon)$, such that if $\varphi(\upsilon, t) = \omega(y_{h1}, \upsilon)$ then $\zeta^{\varphi(\upsilon,t)} = (\rho(y_{h1}, \upsilon), 0)$ and in case that $\hat{y}_{h1} < \bar{y}_{h1}(\upsilon)$, $\omega(y_{h1}, \upsilon)$ is unique and strictly increasing in y_{h1}, equal to 1 if $y_{h1} = \bar{y}_{h1}(\upsilon)$, and equal to 0 if $w = \hat{y}_{h1}$.

Now we turn to the general-equilibrium results that build on Proposition 1. Given a bargaining norm $\zeta(.)$, if $\{(\zeta_t, \phi_t)\}$ be a political-economy equilibrium, then we say that $\{\zeta_t\}$ is a political-economy *policy* and an allocation supported by the equilibrium as a *political-economy allocation*. We say that an allocation $\{(y_{h1t}, y_{l1t}, y_{2t})\}$ is a *stationary allocation* if $(y_{h1t}, y_{l1t}, y_{2t})$ is equal to some (y_{h1}, y_{l1}, y_2) all t, and we use (y_{h1}, y_{l1}, y_2) to represent the stationary allocation.

Proposition 1 has an immediate implication when $\zeta(.)$ is the fiscal-dominance norm.

Lemma 7 *Given the fiscal-dominance norm, there exists a unique political-economy equilibrium $\{(\zeta_t, \phi_t)\}$ and the stationary allocation $(\hat{y}_{h1}, \hat{y}_{l1}, y_2^*)$ is the unique allocation supported by $\{(\zeta_t, \phi_t)\}$.*

The mechanism for determinacy in Lemma 7 has two remarkable features. The first feature is the current or period-t tax response to the asset's future or period-t+1 value. The asset's future value is captured by the future stage-1 consumption x_{t+1} and affects FA's current decision through the relationship $\upsilon = \beta x_{t+1} u'(x_{t+1})$. Depending on parameter values, the Lemma-7 equilibrium can have constant positive inflation and positive lump sum transfers (i.e., negative lump sum taxes) each period. But off the equilibrium path, i.e., in a hypothetical scenario that the economy enters an alternative equilibrium from t, FA is going to adjust current taxes, and in case that the value of υ falls significantly below on-the-equilibrium-path value, FA can even collect positive taxes. Such tax response prevents the hypothetical scenario from materializing. The second feature is that the current stage-1 production does not respond to the asset's future value. The reason is that for FA, the asset's future value only affects its gain $\Delta(\upsilon)$ from reconciliation with CB by way of affecting $i^{\circ}(\upsilon, t)$ (see 29) and, therefore, the current policies corresponding to different future asset values end up with the same stage-1 production).

A couple of questions emerge. If we focus on stationary allocations, what is the set of stationary political-economy allocations? If some allocation in this set is desired for whatever reason, can it be the unique allocation supported the unique political-economy equilibrium given some bargaining norm? To answer these questions, let us draw another implication from Proposition 1.

Lemma 8 *Let Y^e be the set of stationary political-economy allocations. Let $Y^e_{h1} = \{y_{h1} : y_{h1} \in Y^e_{h1}(\upsilon(y_{h1}))\}$, where $\upsilon(y_{h1}) = \beta[\pi_h y_{h1} + \pi_l y_{l1}(y_{h1}) - g]u_1'(\pi_h y_{h1} + \pi_l y_{l1}(y_{h1}) - g)$. Then*

$$Y^e = \{(y_{h1}, y_{l1}, y_2) : y_{h1} \in Y^e_{h1}, y_{l1} = y_{l1}(y_{h1}), y_2 = y_2^*\}.$$

Other than $\hat{y}_{1h}$, any $y_{h1} \in Y^e_{h1}$ is self-dependent because how much a bargaining norm other than the fiscal-dominance norm can raise y_{h1} above $\hat{y}_{1h}$ depends on FA's gain $\Delta(\upsilon)$ from reconciliation and in the general-equilibrium context, $\upsilon(y_{h1})$ is the relevant value of υ. To tackle this self-dependence, we rely on additional conditions on the preferences.

Condition 2 *(a) $-z[u_2''(z) - c_2''(z)] > u_2'(z) - c_2'(z)$ and $-z^{-1}[u_2''(z) - c_2''(z)] > [u_2'''(z) - c_2'''(z)] - [u_2''(z) - c_2''(z)][u_2'(z) - c_2'(z)]^{-1}$; and (b) $z \mapsto z u_1'(z)$ is either constant or strictly monotonic.*

When u_2 and c_2 are power functions, condition 2 (a) can be easily satisfied; condition 2 (b) is standard. Using condition 2 (a), we can show that $\Delta(\upsilon)$ is strictly decreasing in υ. This relationship between υ and $\Delta(\upsilon)$ is intuitive. Indeed, one may see from (28) a partial-equilibrium substitution effect between υ and i_t; that is, a larger υ is accompanied with a smaller i_t so that the value of R_t can maintain the stage-1 production of per type-θ household at $\hat{y}_{1\theta}$. With condition 2 (a), $i^{\circ}(\upsilon, t)$ is strictly decreasing in υ and so is $\Delta(\upsilon)$. Using condition 2 (b), we get a monotonic $y_{h1} \mapsto \upsilon(y_{h1})$. Put together, $y_{h1} \mapsto \Delta(\upsilon(y_{h1}))$ is either constant or strictly monotonic.

To continue, let us first consider the simplest case, i.e., $z u_1'(z)$ is constant in z and so $\Delta(\upsilon(y_{h1}))$ is constant in y_{h1}. In this case, $Y^e_{h1} = Y^e_{h1}(C)$ for some constant C. It follows from Proposition 1 that for each allocation $(y_{h1}, y_{l1}, y_2) \in Y^e_{h1}$, given the generalized Nash bargaining norm $\zeta^{\varphi(.)}$ with $\varphi(\upsilon, t) = \omega(y_{h1}, C)$ all (υ, t), there exists a unique political-economy equilibrium and the allocation is the unique allocation supported by the political economy equilibrium.

Next consider that $zu_1'(z)$ is strictly increasing in z and so $\Delta(\upsilon(y_{h1}))$ is strictly decreasing in y_{h1}. Because FA's payoff loss $\delta(y_{h1})$ from raising y_{h1} above $\hat{y}_{1h}$ is strictly increasing in y_{h1}, there exists a unique $\bar{y}_{h1} > \hat{y}_{l1}$ such that $\Delta(\upsilon(\bar{y}_{h1})) = \delta(\bar{y}_{h1})$ and $[\Delta(\upsilon(y_{h1})) - \delta(y_{h1})](y_{h1} - \bar{y}_{h1}) < 0$ for $y_{h1} \neq \bar{y}_{h1}$, implying $Y^e_{h1} = [\hat{y}_{h1}, \bar{y}_{h1}]$. Again, for each allocation $(y_{h1}, y_{l1}, y_2) \in Y^e_{h1}$, given some generalized Nash bargaining norm $\zeta^{\varphi(.)}$, there exists a unique political-economy equilibrium $\{(\zeta_t, \phi_t)\}$ and (y_{h1}, y_{l1}, y_2) is the unique allocation supported by $\{(\zeta_t, \phi_t)\}$. In the norm $\zeta^{\varphi(.)}$, $\varphi(\upsilon(y_{h1}), t) = \omega(y_{h1}, \upsilon(y_{h1}))$ by Proposition 1. If $\upsilon < \upsilon(y_{h1})$, then let $\varphi(\upsilon, t) = \omega(y_{h1}, \upsilon)$ ($\omega(y_{h1}, \upsilon)$ is well defined because $\Delta(\upsilon)$ is strictly decreasing in υ); if $\upsilon > \upsilon(y_{h1})$, then let $\varphi(\upsilon, t) = 0$. That is, if the asset's future value is low (driving υ below $\upsilon(y_{h1})$), then CB has a bargaining power greater than $\omega(y_{h1}, \upsilon(y_{h1})$ to make y_{h1} the stage-1 production per type-h household; the bargaining power. But if the asset's future value is high (driving υ above $\upsilon(y_{h1})$), then CB has a bargaining power lower than $\omega(y_{h1}, \upsilon(y_{h1}))$ to make $\hat{y}_{1h}$ the stage-1 production per type-h household.

To see why this norm $\zeta^{\varphi(.)}$ leads to determinacy, suppose there is a political-economy equilibrium such that the stage-1 production per type-h household in some period t is not y_{h1}. For this to happen, the stage-1 production y_{h1t+3} per type-h household in period $t+3$ must be different from y_{h1}. Either $y_{h1t+3} < y_{h1}$ or $y_{h1t+3} > y_{h1}$. If the former, then the value of υ relevant for decision making in period $t+2$ is less than $\upsilon(y_{h1})$, implying that the stage-2 production per type-h household is y_{h1} in period $t+2$, and, hence, is so in any $t' < t+2$. So we must have $y_{h1t+3} > y_{h1}$. Then the value of υ relevant for decision making in period $t+2$ is greater than $\upsilon(y_{h1})$, implying that the stage-1 production per type-h household is $\hat{y}_{1h}$ in period $t+2$. But because the value of υ relevant for decision making in period $t+1$ is less than $\upsilon(y_{h1})$, the stage-2 production per type-h household is y_{1h} in period $t+1$, and, hence, is so in t.

Finally, consider that $zu_1'(z)$ is strictly decreasing in z and so $\Delta(\upsilon(y_{h1}))$ is strictly increasing in y_{h1}. This case is somewhat complicate. For there may be multiple solutions to the equation $\Delta(\upsilon(y_{h1})) = \delta(y_{h1})$. That being the case, Y^e_{h1} is a union of multiple closed intervals and determinacy only applies to the closed interval at the left end, denoted $[\hat{y}_{h1}, \bar{y}_{h1}]$. Now for $(y_{h1}, y_{l1}, y_2) \in Y^e$ with $y_{h1} \in [\hat{y}_{l1}, \bar{y}_{h1}]$, again

$\varphi(\upsilon(y_{h1}), t) = \omega(y_{h1}, \upsilon(y_{h1}))$ by Proposition 1. If $\upsilon > \upsilon(y_{h1})$ and $\Delta(\upsilon) < \delta(y_{h1})$, then let $\varphi(\upsilon, t) = 1$. If either $\upsilon > \upsilon(y_{h1})$ and $\Delta(\upsilon) \geq \delta(y_{h1})$ or $\upsilon < \upsilon(y_{h1})$, let $\varphi(\upsilon, t) = \omega(y_{h1}, \upsilon)$. That is, CB's bargaining power is set to make y_{h1} the stage-1 production per type-h household whenever it is feasible. But when it is infeasible to get y_{h1} because of a low asset's future value, CB has all the bargaining power. With this design, there exists $s > 0$ such that when y_{h1} is not the stage-1 production per type-h household in period $t + s$, it must be in period t.

Proposition 2 *There exists $\bar{y}_{1h} \in [\hat{y}_{1h}, y^*_{1h}]$ such that $Y^e \supseteqq Y^{e\prime} = \{(y_{h1}, y_{l1}, y_2) : \hat{y}_{1h} \leq y_{h1} \leq \bar{y}_{1h},\ y_{l1} = y_{l1}(y_{h1}), y^*_2\}$, $Y^e = Y^{e\prime}$ if $z \mapsto zu_1'(z)$ is constant or strictly increasing, and when $\hat{\theta} = h$, $\bar{y}_{1h} > \hat{y}_{1h}$. Moreover, for any $(y_{h1}, y_{l1}, y_2) \in Y^{e\prime}$, given some generalized Nash bargaining norm $\zeta^{\varphi(.)}$, there exists a unique political-economy equilibrium $\{(\zeta_t, \phi_t)\}$ and (y_{h1}, y_{l1}, y_2) is the unique allocation supported by $\{(\zeta_t, \phi_t)\}$.*

7 Discussion

In this section, we discuss how some aggregate shocks may move inflation and nominal interest rate in our model. To this end, we make two modifications of the section-2 model. First, we introduce a liquidity shock by assuming that the utility for each household from consuming x at stage 2 in period t is $\alpha u_2'(x)$ and the realization of α is drawn from a finite set at the start of t with a time-invariant probability. It is straightforward to adapt Definition 1-3 by substituting each periot-t variable (e.g., y_{2t}) with the α-dependent from (e.g., $y_{2t}(\alpha)$) and updating (11) to

$$\phi_{3t}(\alpha) = \beta E \frac{\phi_{t+1}(\alpha')}{1 - \delta_{t+1}(\alpha')} u_1'(\frac{\phi_{t+1}(\alpha')}{1 - \delta_{t+1}(\alpha')}).$$

An interesting alternative to the liquidity shock is the government-expenditure shock; that is, the realization of the government expenditures g in period t is drawn from a finite set at the start of t with a time-invariant probability.

Second, we let the the discount window only only accepts the government bonds. This modification is motivated by the observation that the second-2 model has the nominal interest rate in any political-economy equilibrium. On a general level, the zero nominal interest corresponds to sufficiency in liquidity. In our model, sufficiency pertains to stage-2 trade. To have sufficient liquidity for stage-2 trade, it can be the case that CB must accept private bonds through the discount window in period t, i.e., $\lambda_t > 1$. Let us provide a justification for this modification. Suppose there is a measure zero of households whose producers get sick at stage 3 of a period. Moreover, the central location has many different regions. If a potential lender lives inside the same region as a potential borrower, then it is costless for the former to monitor the health status of the producer of the latter; otherwise, the monitoring cost is high. So the private bond market is segregated. Because the producer gets sick only with probability zero, the nominal interest rate in each segregated private bond market is the same as the rate in the original model. CB does not stay in the same region as private agents. As such, it faces the high monitoring cost, which can endogenize the restriction on the discount window.

To make a focus, we use a parametric example by which the results in last section can be easily adpated. Specifically, let $u_n(x) = \log x$ and $c_n(y) = 0.5y^2$ for $n = 1, 2$. Let $\hat{\theta} = h$. Then it is straightforward to get a positive correlation between positive inflation rates and positive nominal interest rates.

Appendix

Proof of Lemma 1

Fix ς_t satisfying (7)-(9). Then $x_{2t} = y_{2t}$, $u_1'(x_{1t}) \geq \theta c_1'(y_{\theta 1t})$, and $x_{1t} = \sum_\theta \pi_\theta y_{\theta 1t} - g$. By (9), $\delta_t = (\tau_t - g_t)x_{1t}^{-1}$. By the first equality in (8), $\phi_{1t} = (1 - \delta_t)x_{1t}$. By the second equality in (8) and the second equality in (10), $\phi_{2t} = (y_{2t}/\lambda_t)[1 + (1 - \lambda_t)i_t]$. Substituting these values of ϕ_{1t} and ϕ_{2t} into (7), we have

$$\frac{y_{2t}}{\lambda_t} c_2'(y_{2t}) = \frac{\upsilon}{1 + (1 - \lambda_t)i_t}, \tag{30}$$

$$(1 - \delta_t)x_{1t}\theta c_1'(y_{\theta 1t}) = \frac{u_2'(x_{2t})y_{2t}}{\lambda_t}, \tag{31}$$

$$\theta c_1'(y_{\theta 1t}) = \frac{1 + i_t}{(1 - \delta_t)x_{1t}} \frac{\upsilon}{1 + (1 - \lambda_t)\, i_t}, \tag{32}$$

By (31) and (32), $(1 + i_t)\upsilon_t/[1 + (1 - \lambda_t)i_t] = u_2'(x_{2t})y_{2t}/\lambda_t$; then by (30), we get (12). By (30) and (32), we get

$$\theta c_1'(y_{\theta 1t}) = \frac{1 + i_t}{(1 - \delta_t)x_{1t}} \frac{y_{2t}}{\lambda_t} c_2'(y_{2t}). \tag{33}$$

By (33) and (32), we get $\lambda_t = (1 + i_t) y_{2t} c_2'(y_{2t}) [\upsilon_t + i_t y_{2t} c_2'(y_{2t})]^{-1}$. Plugging this λ_t into (33), we get (13).

Proof of Lemma 2

The proof is the same as the proof of 2, with υ replaced by ϕ_{3t} in (11).

Proof of Lemma 3

By the definition of ex ante optimal policy and Lemma 2, a policy $\{\zeta_t\}$ is *ex ante* optimal iff there exists an allocation $\{(y'_{h1t}, y'_{l1t}, y'_{2t})\}$ such that $\{(\zeta_t, y'_{h1t}, y'_{l1t}, y'_{2t})\}$ is a solution to the optimization problem

$$\max \textstyle\sum_t \beta^t [\mu_1(y_{h1t}, y_{l1t}) + \mu_2(y_{2t})]$$

subject to the equilibrium conditions in Lemma 2. Observe that $\{(y''_{h1t}, y''_{l1t}, y''_{2t})\}$ with $(y''_{h1t}, y''_{l1t}, y''_{2t}) = (y^*_{h1}, y^*_{l1}, y^*_2)$ all t is the unique solution to the problem when the constraints are dropped. Because the equilibrium conditions in Lemma 2 hold when $i_t = 0$, $(\tau_t - g)(\sum \pi_\theta y^*_{\theta 1} - g)^{-1} = 1 - \beta$, and $(y_{h1t}, y_{l1t}, y_{2t}) = (y^*_{h1}, y^*_{l1}, y^*_2)$, it follows that the Friedman rule is *ex ante* optimal and $\{(y''_{h1t}, y''_{l1t}, y''_{2t})\}$ is the *ex ante* optimal allocation.

Proof of Lemma 4

Let κ_t be per type-h household stage-1 borrowing from type-l households in the unit of stage-1 goods. Notice that every household carries $\lambda_t L_t$ units of money at the end of stage 1 and a type-h household only holds money at the end of stage 1. Thus $P_{1t}(y_{h1t} - \tau_t + \kappa_t) = \lambda_t L_t$. And, because at stage 1 per household nominal lending to the government is $(1 - \lambda_t) L_t$, a type-l household nominal lending to the government is $(1 - \lambda_t) L_t \pi_l^{-1}$, implying $P_{1t}(y_{l1t} - \tau_t - \pi_h \kappa_t \pi_l^{-1}) - (1 - \lambda_t) L_t \pi_l^{-1} = \lambda_t L_t$. This and $P_{1t}(y_{h1t} - \tau_t + \kappa_t) = \lambda_t L_t$ give rise to $y_{h1t} + \kappa_t = y_{l1t} - \pi_h \kappa_t \pi_l^{-1} - (1 - \lambda_t) L_t P_{1t}^{-1} \pi_l^{-1}$. Using $L_t P_{1t}^{-1} = (1 - \delta_t) x_{1t}$, we have

$$\kappa_t = \pi_l (y_{l1t} - y_{h1t}) - (1 - \lambda_t)(1 - \delta_t) x_{1t}. \tag{34}$$

When reaching stage 3, all households carry the same amount of money, each type-h household has $(1 + i_t)\upsilon[1 + (1 - \lambda_t)\, i_t]^{-1}[x_{1t}(1 - \delta_t)]^{-1}\kappa_t$ private debt in the unit of stage-3 goods, only type-l households hold government bonds, and government bonds are worth $(1-\lambda_t)(1+i_t)[1+(1-\lambda_t)\,i_t]^{-1}\upsilon$ in the unit of stage-3 goods. At the end of stage 3, each household net worth is υ in the unit of stage-3 goods. To reach this net worth, a type-h household produces

$$y_{h3t} = \frac{(1-\lambda_t)(1+i_t)\upsilon}{1+(1-\lambda_t)i_t} + \frac{1+i_t}{1+(1-\lambda_t)\,i_t}\frac{\upsilon\kappa_t}{x_{1t}(1-\delta_t)} \tag{35}$$

at stage 3, and, correspondingly, a type-l household consumes $x_{l3t} = (\pi_h/\pi_l)y_{3ht}$. So

$$y_{h3t} = \frac{1+i_t}{1+(1-\lambda_t)\,i_t}\frac{\upsilon\pi_l(y_{l1t}-y_{h1t})}{x_{1t}(1-\delta_t)} = \pi_l(y_{l1t}-y_{h1t})hc_1'(y_{h1t}),$$

where the first equality uses (34) into (35), and the second equality uses (30).

Proof of Lemma 6

Let $i^\circ = i_t^\circ(\upsilon)$ and $\tau^\circ = \tau_t^\circ$. Because $i_t = i^\circ$ is a best response to $\tau_t = \tau^\circ$ and $\hat{y}_{\theta 1}$ is the stage-1 production in the temporary allocation supported by $(\upsilon, (\tau^\circ, i^\circ))$, the equilibrium condition (13) implies

$$\hat{x}_{1t}\theta c_1'(\hat{y}_{\theta 1})(1-\delta^\circ) = \upsilon_t + i^\circ y_2(i^\circ)c_2'(y_2(i^\circ)). \tag{36}$$

Write $y_{\theta 1}(i)$ as $y_{\theta 1}(i)$ and let $\varkappa(i) = [\sum \pi_\theta y_{\theta 1}(i) - g]\theta c_1'(y_{\theta 1}(i))$. By (13) and (36), $y_{\theta 1}(i)$ satisfies $\varkappa(i)\theta c_1'(y_{\theta 1}(i))(1-\delta^\circ) - \hat{x}_{1t}\theta c_1'(\hat{y}_{\theta 1})(1-\delta^\circ) = iy_2(i)c_2'(y_2(i)) - i^\circ y_2(i^\circ)c_2'(y_2(i^\circ))$, implying

$$\varkappa'(i) = \frac{y_2(i)c_2'(y_2(i)) + iy_2'(i)c_2'(y_2(i)) + iy_2(i)y_2'(i)c_2''(y_2(i))}{1-\delta^\circ}. \tag{37}$$

By definition, $\varkappa'(i) = hc_1'(y_{h1}(i))\sum \pi_\theta y_{\theta 1}'(i) + x_1(i)hc_1''(y_{h1}(i))y_{h1}'(i)$; by $hc_1'(y_{h1}(i)) = lc_1'(y_{l1}(i))$, $c_1''(y_{h1}(i))c_1'(y_{l1}(i))y_{h1}'(i) = c_1''(y_{l1}(i))c_1'(y_{h1}(i))y_{l1}'(i)$. Thus

$$\varkappa'(i) = hc_1'(y_{h1}(i))[\pi_h + \pi_l\frac{c_1''(y_{h1}(i))}{c_1'(y_{h1}(i))}\frac{c_1'(y_{l1}(i))}{c_1''(y_{l1}(i))}y_{h1}'(i) + x_1(i)hc_1''(y_{h1}(i))y_{h1}'(i). \tag{38}$$

Now plug in the value of $\varkappa'(i)$ in (38) and the value of $1-\delta^\circ$ obtained from (36) into (37), and then evaluate (37) at $i = i^\circ$; letting $y_2' = y_2'(i^\circ)$ and $y_2 = y_2'(i^\circ)$, the result is

$$\frac{-i^\circ y_2'c_2'(y_2)[\upsilon + i^\circ y_2c_2'(y_2)]}{y_2c_2'(y_2) + i^\circ y_2'c_2'(y_2) + i^\circ y_2y_2'c_2''(y_2)} = A. \tag{39}$$

By (12), $i^\circ = [u_2'(y_2) - c_2'(y_2)]/c_2'(y_2)$ and $y_2' = c_2'(y_2)[u_2''(y_2) - (1+i^\circ)c_2''(y_2)]^{-1}$. Plugging these values of i° and y_2' into (39), we get (27).

Proof of Proposition 1

It suffices to consider $\hat{\theta} = h$. First we show that $i_t = 0$ if $\zeta_t = (\tau_t, i_t) \in \Xi^e(\upsilon, t)$. Suppose by contradiction that $i_t > 0$. Let $(y_{h1t}, y_{l1t}, y_{2t})$ be the temporary allocation admitted by ζ_t. Because $i_t > 0$, $y_{2t} \neq y_2^*$. We claim that some $\zeta_t' = (\tau_t', i_t') \in \Xi^e(\upsilon, t)$ with $i_t' = 0$ admits the temporary allocation $(y_{h1t}, y_{l1t}, y_{2t}')$ with $y_{2t}' = y_2^*$. Then by Lemma 4, FA's payoff from ζ_t' is greater than ζ_t, a contradiction. To verify the claim, it suffice to pick τ_t' such that (13) holds when δ_t is replaced with $\delta_t' = (\tau_t' - g)x_{1t}^{-1}$ and i_t is replaced with i_t'.

Next we show that $Y^e(\upsilon, t) = \{(y_{h1}, y_{l1}, y_2) : \hat{y}_{h1} \le y_{h1} \le \bar{y}_{h1}(\upsilon),\ y_{l1} = y_{l1}(y_{h1}),\ y_2 = y_2^*\}$. For this, it suffices to note that by condition 1, $y_{h1} \mapsto \eta_1(y_{h1}, y_{l1}(y_{h1}), h)$ is strictly concave and, hence, $y_{h1} \mapsto \delta(y_{h1})$ is strict convex.

Now we turn to the part for generalized Nash bargaining norm. Let $\tau^\circ = \tau^\circ(\upsilon, t)$, $i^\circ = i^\circ(\upsilon, t)$, $\zeta^\circ = (\tau^\circ, i^\circ)$, and $y_2^\circ = y_2(i^\circ)$; then $V(\zeta^\circ, \upsilon, h) = \eta(\hat{y}_{1h}, \hat{y}_{1l}, h) + \mu_2(y_2^\circ)$ and $W(\zeta^\circ, \upsilon) = \mu_1(\hat{y}_{1h}, \hat{y}_{1l}) + \mu_2(y_2^\circ)$. For $z \in Y_{h1}^e(\upsilon)$, let $\zeta(z) = (\rho(z, \upsilon), 0)$; then $V(\zeta(z), \upsilon, h) = \eta(z, y_{l1}(z), h) + \mu_2(y_2^*)$ and $W(\zeta(z), \upsilon) = \mu_1(z, y_{l1}(z)) + \mu_2(y_2^*)$. Let $\Delta_W(z) = W(\zeta(z), \upsilon) - W(\zeta^\circ, \upsilon)$ and $\Delta_V(z) = V(\zeta(z), \upsilon, h) - V(\zeta^\circ, \upsilon, h)$. Using (17), $\zeta(z)$ is the policy specified by the generalized Nash bargaining norm $\zeta^{\varphi(.)}$ if $\varphi(\upsilon, t)$ is equal to some ω satisfying

$$-\omega \Delta_W'(z)\Delta_V(z) = (1 - \omega)\Delta_V'(z)\Delta_W(z). \tag{40}$$

By $\Delta_W'(\hat{y}_{1h}) = 0$, (40) implies $\omega = 0$ when $z = \hat{y}_{1h}$; by $\Delta_V(\bar{y}_{1h}(\upsilon)) = 0$, (40) implies $\omega = 1$ when $z = \bar{y}_{1h}(\upsilon)$. For $z \in (\hat{y}_{1h}, \bar{y}_{1h}(\upsilon))$, there exists a unique ω satisfying (40). Therefore, $\omega(z, \upsilon)$ is well defined and unique for any $z \in Y_{h1}^e(\upsilon)$. Because $\Delta_W'(z)/\Delta_V'(z)$ is decreasing in z and $\Delta_W(z)/\Delta_V(z)$ is increasing, $\omega(z, \upsilon)$ is increasing in z.

Proof of Proposition 2

Here we verify three assertions made in the argument receding Proposition 2. First, we verify that $\Delta(\upsilon)$ is strictly decreasing in υ. Write (27) as $\upsilon + A = f(z)$, where $f(z) = -z[u_2'(z) - c_2'(z)]^2[u_2'(z) - c_2'(z)]^{-1} - Az[u_2''(z) - c_2''(z)][u_2'(z) - c_2'(z)]^{-1}$. By Lemma 6, it suffices to verify $f'(z) > 0$. Let

$$\varrho(z) = -\frac{u_2''(z) - c_2''(z)}{u_2'(z) - c_2'(z)} + \frac{z[u_2''(z) - c_2''(z)]^2 - z[u_2'''(z) - c_2'''(z)][u_2'(z) - c_2'(z)]}{[u_2'(z) - c_2'(z)]^2}.$$

Then $f'(z) = -[u_2'(z) - c_2'(z)] - z[u_2''(z) - c_2''(z)] - A\varrho(z)$ and by condition 2 (a), $f'(z) > 0$.

Next, we verify that in case (iii), Y_{h1}^e is a union of multiple closed intervals and the closed interval at the left end takes the form $[\hat{y}_{h1}, \bar{y}_{h1}]$. Let $\tilde{\Delta} = \mu_2(y_2^*)$ and let $\tilde{y}_{h1}$ be defined by $\eta(\hat{y}_{h1}, \hat{y}_{l1}, h) - \tilde{\Delta} = \eta(\tilde{y}_{h1}, y_{l1}(\tilde{y}_{h1}), h)$. For $z \in [\hat{y}_{h1}, \tilde{y}_{h1}]$, let $a(z)$ be defined by

$$\eta(\hat{y}_{h1}, \hat{y}_{l1}, h) - \Delta(\upsilon(z)) = \eta(a(z), y_{l1}(a(z)), h). \tag{41}$$

for $z \in [\hat{y}_{h1}, \tilde{y}_{h1}]$. Because $\Delta(z) \geq \Delta(\hat{y}_{h1})$, $a(z) \geq \hat{y}_{h1}$. Because $\tilde{\Delta} \geq \Delta(z)$, $a(z) \leq \tilde{y}_{h1}$. Because $a(z) > a(z')$ for $z > z'$, $a(.)$ is a monotonic mapping from $[\hat{y}_{h1}, \tilde{y}_{h1}]$ to $[\hat{y}_{h1}, \tilde{y}_{h1}]$ and, hence has a least fixed point, which is $\bar{y}_{h1}$.

Now we verify that in case (iii), for $(y_{h1}, y_{l1}, y_2) \in Y^e$ with $y_{h1} \in [\hat{y}_{l1}, \bar{y}_{h1}]$, given the bargaining norm, there exists $s > t$ such that when y_{h1s} is not the stage-1 production y_{h1} per type-h household in period s, it must be in period t. Pick an arbitrarily large s with $y_{h1s} \neq y_{h1}$. First suppose either $\upsilon(y_{h1s}) > \upsilon(y_{h1})$ and $\Delta(\upsilon(y_{h1s})) \geq \delta(y_{h1})$ or $\upsilon(y_{h1s}) < \upsilon(y_{h1})$. Then the value of υ relevant for decision making in period $s-1$ satisfies $\varphi(\upsilon, s-1) = \omega(y_{h1}, \upsilon)$, implying that the stage-1 production per type-h household is y_{h1} in period $s-1$, and, hence, is so in any $t' < s$. To complete the proof, suppose $\upsilon(y_{h1s}) > \upsilon(y_{h1})$ and $\Delta(\upsilon(y_{h1s})) < \delta(y_{h1})$. Then the value of υ relevant for decision making in period $s-1$ satisfies $\varphi(\upsilon, s) = 1$. Write y_{h1s} as z_s so the stage-1 production per type-h household in period $s-1$ is $z_{s-1} = a(z_s)$, where a is the mapping defined by (41). By induction, we obtain an increasing sequence $\{z_{s-l}\}_{l\geq 1}$ by $z_{s-l-1} = a(z_l)$. Let the sequence $\{z'_{s-l}\}_{l\geq 1}$ be defined by $z'_s = \hat{y}_{h1}$ and $z'_{s-l-1} = a(z'_l)$. Notice that $z'_{s-l} \geq z_{s-l}$ all l and $\lim z_{s-l} = \lim z'_{s-l} = \bar{y}_{h1}$. Therefore, there exists some l_0 which does not depend on the values of z_s and s such that $z_{s-l} > y_{h1}$ whenever $l > l_0$. That is, the value of υ relevant for decision making in period $s - l_0$ satisfies $\varphi(\upsilon, s - l_0) = \omega(y_{h1}, \upsilon)$, implying that the stage-1 production per type-h household is y_{h1} in period $s - l_0$, and, hence, is so in any $t' < s - l_0$.

References

[1] Alesina, Alberto and Guido Tabellini, Rules and Discretion with Non-coordinated Monetary and Fiscal Policy, *Economic Inquiry* 25 (1987), 619-30.

[2] Albanesi, Stefania, Inflation and Inequality, *Journal of Monetary Economics* 54 (2007), 1088–1114.

[3] Arouba, Boragan and Sanjay K. Chugh, Optimal Fiscal and Monetary Policy When Money is Essential, *Journal of Economic Theory* 145 (2010), 1618-1647.

[4] Barro, Robert J. and Gordon, David B, A Positive Theory of Monetary Policy in a Natural Rate Model. *Journal of Political Economy* 91(1983), 589-610.

[5] Bassetto, Marco and Thomas J. Sargent, Shotgun Wedding: Fiscal and Monetary Policy, *Annual Review of Economics* 12 (2020), 659-90.

[6] Barthélemy, Jean and Guillaume Plantin, Fiscal and Monetary Regimes: A Strategic Approach, *European Economic Review* 54 (2010), 855–879.

[7] Camous, Antoine and Dmitry Matveev, The Central Bank Strikes Back! Credibility of Monetary Policy under Fiscal Influence, *The Economic Journal*, 133 (2023), 1-29.

[8] Dixit, Avinash and Luisa Lambertini, Interactions of Commitment and Discretion in Monetary and Fiscal Policies, *American Economic Review* 2003 (93),1522–1542.

[9] Dolmas, Jim, Gregory W. Huffman and Mark A. Wynne, Inflation and Central Bank Independance, *Canadian Journal of Economics* 2000 (33), 271-287.

[10] Eggertsson, Gauti and Eric Le Borgne, A Political Agency Theory of Central Bank Independence, *Journal of Money, Credit and Banking* 42 (2010), 647-677.

[11] Fragetta, Matteo and Tatiana Kirsanova, Strategic Monetary and Fiscal Policy Interactions: An Empirical Investigation, *European Economic Review* 54 (2010), 855–879.

[12] Friedman, Milton, Should There Be an Independent Monetary Authority? in Leland B. Yeager (Ed.), In Search of a Monetary Constitution, Harvard University Press, 1962.

[13] Friedman, Milton, The Optimal Quantity of Money and Other Essays, Chicago: Aldine, 1969.

[14] Grembi, Veronica, Tommaso Nannicini and Ugo Troiano, Do Fiscal Rules Matter?, *American Economic Journal: Applied Economics* 2016, 8(3): 1–30.

[15] Helpman, Elhanan and Sadka, Efraim, Optimal Financing of the Government's Budget: Taxes, Bonds, or Money? *American Economic Review*, 69 (1979), 152-160.

[16] Herrenbrueck, Lucas, Frictional Asset Markets and the Liquidity Channel of Monetary Policy, *Journal of Economic Theory* 181 (2019), 82-120.

[17] Jin, Gu, and Tao Zhu, Heterogeneity, decentralized trade, and the long-run real effects of inflation, *Journal of Economic Theory* 201 (2022), 105439.

[18] Kydland, Finn E. and Prescott, Edward C, Rules Rather Than Discretion: The In consistency of Optimal Plans, *Journal of Political Economy* 85 (1977), 473-91.

[19] Lagos, Ricardo and Randall Wright, A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy* 113 (2005), 463-484.

[20] Leeper, Eric M., Campbell Leith and Ding Liu, Optimal Time-Consistent Monetary, Fiscal and Debt Maturity Policy, *Journal of Monetary Economics* 117 (2021), 600–617.

[21] Lucas, Robert E, Inflation and Welfare, *Econometrica* 68 (2000), 247–74.

[22] Martin, Fernando M., On the Joint Determination of Fiscal and Monetary Policy, *Journal of Monetary Economics* 58 (2011) 132–145.

[23] Meltzer, Allan H., *A History of the Federal Reserve*, Volume 2, Book 1, 1951–1969, University of Chicago Press, 2009.

[24] Miller, David S., Commitment versus Discretion in a Political Economy Model of Fiscal and Monetary Policy Interaction, *Journal of Monetary Economics* 84 (2016), 17–29.

[25] Molico, Miguel, The Distribution of Money and Prices in Search Equilibrium, *International Economic Review* 47 (2006), 701-722.

[26] Niemann, Stefan, Dynamic Monetary-Fiscal Interactions and the Role of Monetary Conservatism, *Journal of Monetary Economics* 58 (2011), 234–47.

[27] Niemann, Stefan, Paul Pichler and Gerhard Sorger, Central Bank Independence and the Monetary Instrument Problem, *International Economic Review* 54 (2013), 1031-1055.

[28] Nowotny, Ewald, Inflation and Taxation: Reviewing The Macroeconomic Issues, *Journal of Economic Literature* 18 (1980): 1025–49.

[29] Persson, Torsten and Guido Tabellini, Designing Institutions for Monetary Stability, *Carnegie-Rochester Conference Series on Public Policy* 39 (1993), 53-84.

[30] Rocheteau, Guillaume, Randall Wright, and Sylvia Xiaolin Xiao, Open Market Operations, *Journal of Monetary Economics* 98 (2018) 114–128.

[31] Rogoff, Kenneth, The Optimal Degree of Commitment to an Intermediate Monetary Target, *Quarterly Journal of Economics* 100 (1985), 1169-1189.

[32] Sargent, Thomas J., *Macroeconomic Theory*, Second Edition, Academic Press 1987.

[33] Sargent, Thomas J. and Neil Wallace, Some Unpleasant Monetarist Arithmetic, *Federal Reserve Bank of Minneapolis*, *Quarterly Review* 5 (1981), 1-17.

[34] Sims, Christopher, A Review of "Monetary Policy Rules," by John B. Taylor. *Journal of Economic Literature* 39, no. 2 (2001): 562–66.

[35] Schmitt-Grohe, Stephanie and Martin Uribe, The Optimal Rate of Inflation, Editor(s): Benjamin M. Friedman, Michael Woodford, *Handbook of Monetary Economics*, Volume 3, 2010, 653-722.

[36] Tabellini, Guido, Money Debt and Deficits in a Dynamic Game, *Journal of Economic Dynamics and Control* 10 (1986), 427-442.

[37] Tabellini, Guido, Designing institutions for monetary stability, *Carnegie-Rochester Conference Series on Public Policy*, 39 (1993), 53-84.

[38] Walsh, Carl E., Optimal Contracts for Central Bankers, *American Economic Review* 85 (1995), 150-167.

[39] Williamson, Stephen D., Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach, *American Economic Review* 102 (2012), 2570-2605.